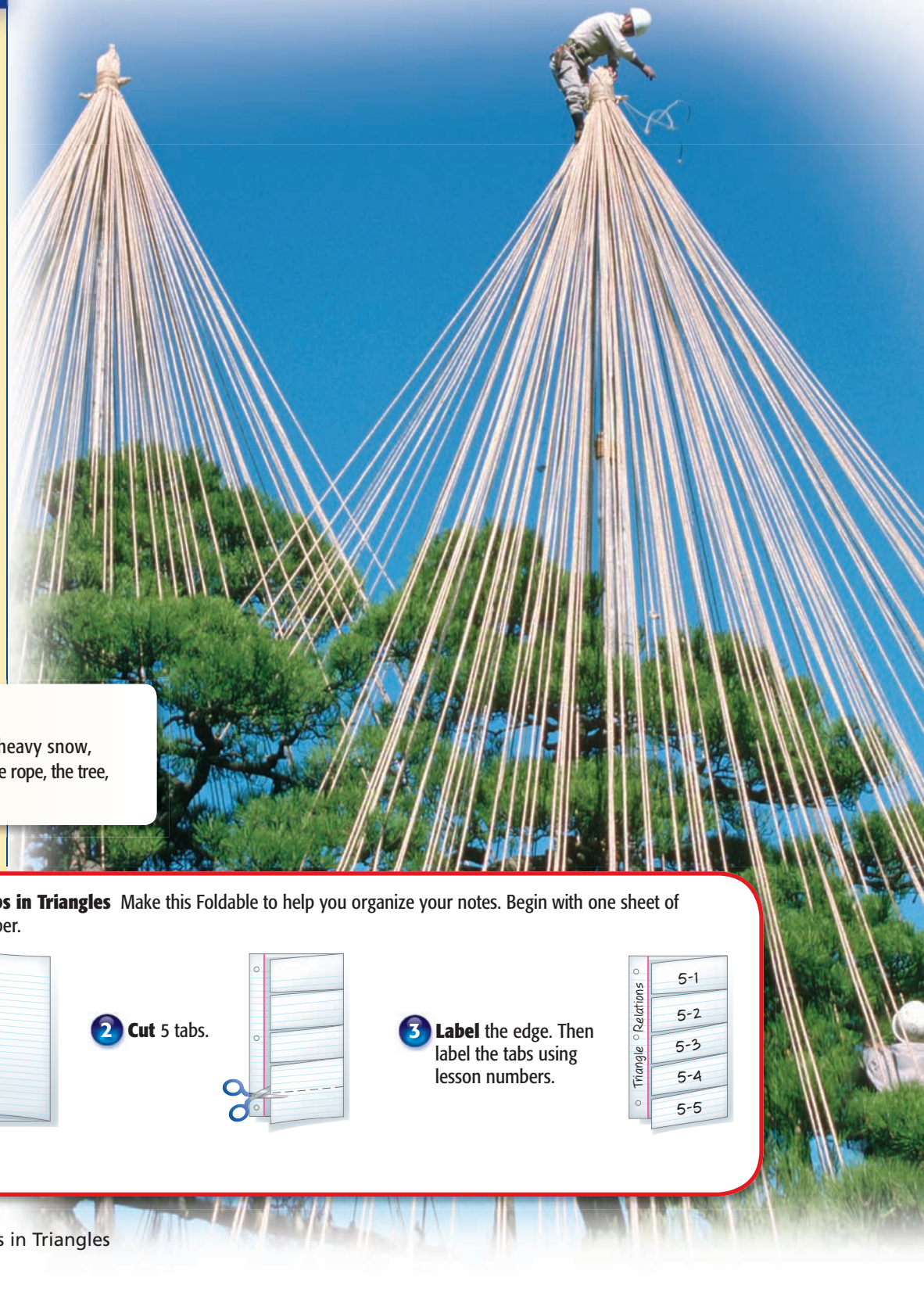
CHAPTER 5

Relationships in Triangles



BIG Ideas

- **Standard 6.0** Students know and are able to use the triangle inequality theorem.
- **Standard 12.0** Students find and use measures of sides and of interior and exterior angles of triangles and polygons to classify figures and solve problems. (Key)
- **Standard 13.0** Students prove relationships between angles in polygons by using properties of complementary, supplementary, vertical, and exterior angles.

Key Vocabulary

perpendicular bisector (p. 269)
median (p. 271)
altitude (p. 272)
indirect proof (p. 288)

Real-World Link

Gardening To protect a tree from heavy snow, gardeners tie a rope to each branch. The rope, the tree, and the ground form a triangle.

FOLDABLES™ Study Organizer

Relationships in Triangles Make this Foldable to help you organize your notes. Begin with one sheet of notebook paper.

1. **Fold** lengthwise to the holes.

2. **Cut** 5 tabs.

3. **Label** the edge. Then label the tabs using lesson numbers.

GET READY for Chapter 5

Diagnose Readiness You have two options for checking Prerequisite Skills.

Option 2

MathOnline Take the Online Readiness Quiz at ca.geometryonline.com.

Option 1

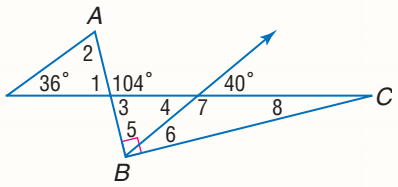
Take the Quick Check below. Refer to the Quick Review for help.

QUICKCheck

Find the coordinates of the midpoint of a segment with the given endpoints. (Lesson 1-3)

1. $A(-12, -5)$, $B(4, 15)$
2. $C(-22, -25)$, $D(10, 10)$
3. **MAPS** The coordinates of Springville are $(-15, 25)$, and the coordinates of Pickton are $(5, -16)$. Hatfield is located midway between the two cities. Find the coordinates of Hatfield. (Lesson 1-3)

Find the measure of each numbered angle if $\overline{AB} \perp \overline{BC}$. (Lesson 1-5)



4. $\angle 1$
5. $\angle 2$
6. $\angle 3$
7. $\angle 4$
8. $\angle 5$
9. $\angle 6$
10. $\angle 7$
11. $\angle 8$

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment. If a valid conclusion is possible, state it. Otherwise, write *no conclusion*. (Lessons 4-4 and 4-5)

12. (1) If the three sides of one triangle are congruent to the three sides of a second triangle, then the triangles are congruent.
(2) $\triangle ABC$ and $\triangle PQR$ are congruent.

QUICKReview

EXAMPLE 1

Find the coordinates of the midpoint of the segment with endpoints $Y(9, 4)$ and $Z(13, 20)$.

Let $(x_1, y_1) = (9, 4)$ and let $(x_2, y_2) = (13, 20)$.

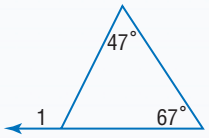
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ Midpoint Formula

$= M\left(\frac{9 + 13}{2}, \frac{4 + 20}{2}\right)$ Substitution

$= M(11, 12)$ Simplify.

EXAMPLE 2

Find $m\angle 1$.



$m\angle 1 = 47 + 67$ Exterior Angle Theorem

$m\angle 1 = 114$ Simplify.

EXAMPLE 3

Determine whether a valid conclusion can be reached from the two true statements using the Law of Detachment. If a valid conclusion is possible, state it. Otherwise, write *no conclusion*.

(1) If two angles make a linear pair, then they are supplementary.

(2) $\angle A$ and $\angle B$ make a linear pair.

A valid conclusion can be reached from the above two statements. $\angle A$ and $\angle B$ are supplementary.

EXPLORE 5-1

Geometry Lab

Bisectors, Medians, and Altitudes of Triangles

Standard 16.0 **Students perform basic constructions with a straightedge and compass, such as angle bisectors, perpendicular bisectors,** and the line parallel to a given line through a point off the line. (Key)

There are four special segments in triangles. You can use the constructions you have learned for midpoints, perpendicular segments, and angle bisectors to construct the special segments in triangles.

CONSTRUCTION 1 Perpendicular Bisector

Concepts in Motion
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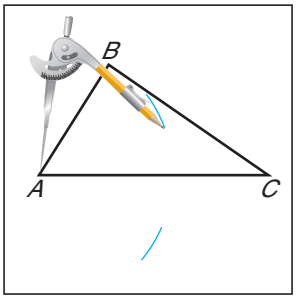
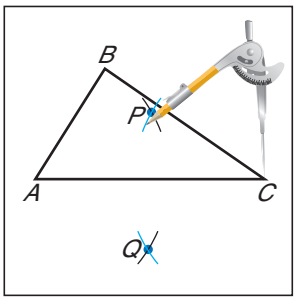
Construct the perpendicular bisector of a side of a triangle.

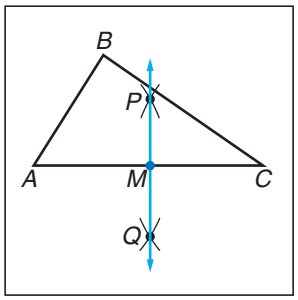
Step 1 Draw a triangle like ABC. Adjust the compass to an opening greater than $\frac{1}{2}AC$. Place the compass at vertex A, and draw an arc above and below $\overline{AC}$.



Step 2 Using the same compass settings, place the compass at vertex C. Draw an arc above and below $\overline{AC}$. Label the points of intersection of the arcs P and Q.



Step 3 Use a straightedge to draw $\overleftrightarrow{PQ}$. Label the point where $\overleftrightarrow{PQ}$ intersects $\overline{AC}$ as M.

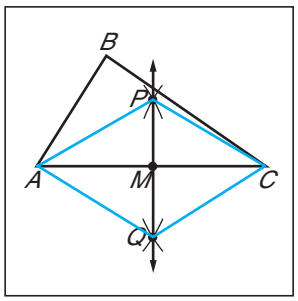


This procedure works for bisecting any segment, not just a side of a triangle.

Verify the construction.

Given: $\triangle ABC$

Prove: $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AC}$ at M.



Paragraph Proof: $\overline{AP} \cong \overline{CP} \cong \overline{AQ} \cong \overline{CQ}$ because the arcs were drawn with the same compass setting. $\overline{AC} \cong \overline{AC}$ by the Reflexive Property. Thus, $\triangle APC \cong \triangle AQC$ by SSS. By CPCTC, $\angle PCA \cong \angle QCA$. $\overline{MC} \cong \overline{MC}$ by the Reflexive Property. Therefore $\triangle MPC \cong \triangle MQC$ by SAS. Then $\angle PMC \cong \angle QMC$ by CPCTC. Since a linear pair of congruent angles are right angles, $\angle PMC$ and $\angle QMC$ are right angles. So $\overleftrightarrow{PQ} \perp \overline{AC}$. $\overline{PM} \cong \overline{PM}$ by the Reflexive Property. $\angle PMA \cong \angle PMC$ since perpendicular lines form four right angles and all right angles are congruent. Thus, $\triangle PMA \cong \triangle PMC$ by HL and $\overline{MA} \cong \overline{MC}$ by CPTPC. Therefore $\overleftrightarrow{PQ}$ bisects $\overline{AC}$ by the definition of bisector.

Analyze the Results

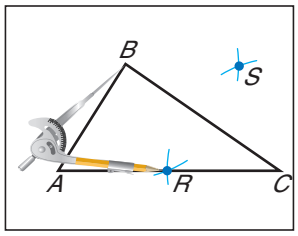
1. Construct the perpendicular bisectors for the other two sides of ABC.
2. What do you notice about the perpendicular bisectors?

A *median* of a triangle is a segment with endpoints that are a vertex of the triangle and the midpoint of the side opposite the vertex. You can construct a median of a triangle using the construction of the midpoint of a segment.

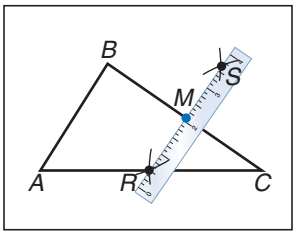
CONSTRUCTION 2 Median

Construct the median of a triangle.

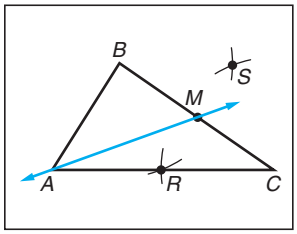
Step 1 Draw intersecting arcs above and below $\overline{BC}$. Label the points of intersection R and S.



Step 2 Use a straightedge to find the point where $\overline{RS}$ intersects $\overline{BC}$. Label the midpoint M.



Step 3 Draw a line through A and M. $\overline{AM}$ is a median of $\triangle ABC$.



ANALYZE THE RESULTS

3. Construct the medians of the other two sides.

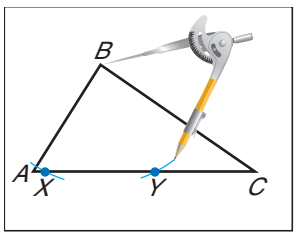
4. What do you notice about the medians of a triangle?

An *altitude* of a triangle is a segment from a vertex of the triangle to the line containing the opposite side and perpendicular to the line containing that side.

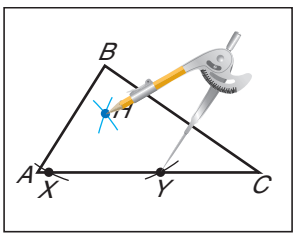
CONSTRUCTION 3 Altitude

Construct the altitude of a triangle.

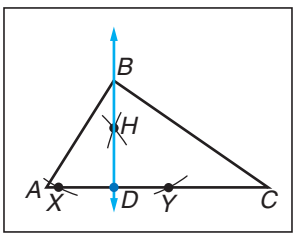
Step 1 Place the compass at vertex B and draw two arcs intersecting $\overleftrightarrow{AC}$. Label the points where the arcs intersect the side X and Y.



Step 2 Adjust the compass to an opening greater than $\frac{1}{2}XY$. Place the compass on point X and draw an arc above $\overline{AC}$. Using the same setting, place the compass on point Y and draw another arc above $\overline{AC}$. Label the point of intersection H.



Step 3 Use a straightedge to draw $\overleftrightarrow{BH}$. Label the point where $\overleftrightarrow{BH}$ intersects $\overline{AC}$ as D. $\overline{BD}$ is an altitude of $\triangle ABC$ and is perpendicular to $\overline{AC}$.



Analyze the Results

5. Construct the altitudes to the other two sides. (*Hint*: You may need to extend the lines containing the sides of your triangle.)
6. What observation can you make about the altitudes of your triangle?

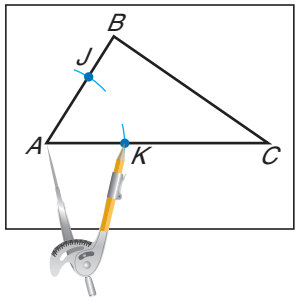
An *angle bisector* of a triangle is a line containing a vertex of a triangle and bisecting that angle.

CONSTRUCTION 4 Angle Bisector

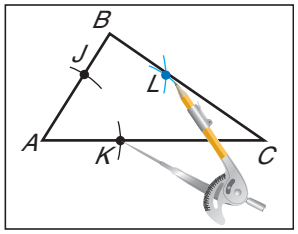
Concepts in MOtion
Animation ca.geometryonline.com

Construct an angle bisector of a triangle.

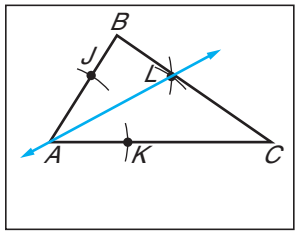
Step 1 Place the compass on vertex A, and draw an arc through $\overline{AB}$ and an arc through $\overline{AC}$. Label the points where the arcs intersect the sides as J and K.



Step 2 Place the compass on J, and draw an arc. Then place the compass on K and draw an arc intersecting the first arc. Label the intersection L.



Step 3 Use a straightedge to draw $\overleftrightarrow{AL}$. $\overleftrightarrow{AL}$ is an angle bisector of $\triangle ABC$.



Analyze the Results

7. **MAKE A CONJECTURE** Predict a relationship involving the angle bisectors of a triangle.
8. Construct the angle bisectors for the other two angles of your $\triangle ABC$. How do the results compare to your conjecture? Explain.

Extend

9. Repeat the four constructions for each type of triangle.
   - **a.** obtuse scalene
   - **b.** right scalene
   - **c.** acute isosceles
   - **d.** obtuse isosceles
   - **e.** right isosceles
   - **f.** equilateral
10. Where are the points of intersection of the lines for an acute triangle?
11. In an obtuse triangle, where are the points of intersection of the lines?
12. Where are the points of intersection of the lines for a right triangle?
13. Under what circumstances do the special lines of triangles coincide with each other?

5-1 Bisectors, Medians, and Altitudes

Main Ideas

- Identify and use perpendicular bisectors and angle bisectors in triangles.
- Identify and use medians and altitudes in triangles.

Standard 12.0 **Students find and use measures** of sides and of interior and exterior angles **of triangles** and polygons **to** classify figures and **solve problems.** (Key)

New Vocabulary

perpendicular bisector
concurrent lines
point of concurrency
circumcenter
incenter
median
centroid
altitude
orthocenter

GET READY for the Lesson

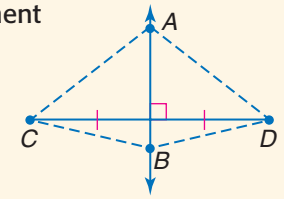
Acrobats and jugglers often balance objects when performing. These skilled artists need to find the center of gravity for each object or body position in order to keep balanced. The center of gravity for any triangle can be found by drawing the *medians* of a triangle and locating the point where they intersect.

Perpendicular Bisectors and Angle Bisectors The first construction you made in the Geometry Lab on pages 266–268 was the perpendicular bisector of a side of a triangle. A **perpendicular bisector** of a side of a triangle is a line, segment, or ray that passes through the midpoint of the side and is perpendicular to that side. Perpendicular bisectors of segments have some special properties. Two of those properties are stated in Theorems 5.1 and 5.2.

THEOREMS *Points on Perpendicular Bisectors*

5.1 Any point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment.

Example: If $\overline{AB} \perp \overline{CD}$ and $\overline{AB}$ bisects $\overline{CD}$, then $AC = AD$ and $BC = BD$.



5.2 Any point equidistant from the endpoints of a segment lies on the perpendicular bisector of the segment.

Example: If $AC = AD$, then A lies on the perpendicular bisector of $\overline{CD}$.
If $BC = BD$, then B lies on the perpendicular bisector of $\overline{CD}$.

You will prove Theorems 5.1 and 5.2 in Check Your Progress 1 and Exercise 23, respectively.

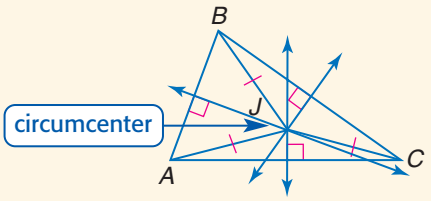
Recall that a locus is the set of all points that satisfy a given condition. A perpendicular bisector of a given segment can be described as the locus of points in a plane equidistant from the endpoints of the given segment.

Since a triangle has three sides, there are three perpendicular bisectors in a triangle. The perpendicular bisectors of a triangle intersect at a common point. When three or more lines intersect at a common point, the lines are called **concurrent lines**, and their point of intersection is called the **point of concurrency**. The point of concurrency of the perpendicular bisectors of a triangle is called the **circumcenter**.

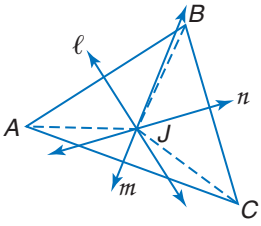
THEOREM 5.3 **Circumcenter Theorem**

The circumcenter of a triangle is equidistant from the vertices of the triangle.

Example: If J is the circumcenter of $\triangle ABC$, then $AJ = BJ = CJ$.



Proof **Theorem 5.3**



Given: ℓ, m, and n are perpendicular bisectors of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$, respectively.

Prove: $AJ = BJ = CJ$

Paragraph Proof:

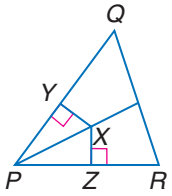
Since J lies on the perpendicular bisector of $\overline{AB}$, it is equidistant from A and B. By the definition of equidistant, $AJ = BJ$. The perpendicular bisector of $\overline{BC}$ also contains J. Thus, $BJ = CJ$. By the Transitive Property of Equality, $AJ = CJ$. Thus, $AJ = BJ = CJ$.

Another special line, segment, or ray in triangles is an angle bisector.

Review Vocabulary

Angle Bisector a ray that divides an angle into two congruent angles (Lesson 1-4)

EXAMPLE **Use Angle Bisectors**



1 **Given:** $\overline{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$.

Prove: $\overline{XY} \cong \overline{XZ}$

Proof:

Statements	Reasons
1. $\overline{PX}$ bisects $\angle QPR$, $\overline{XY} \perp \overline{PQ}$, and $\overline{XZ} \perp \overline{PR}$.	**1.** Given
2. $\angle YPX \cong \angle ZPX$	**2.** Definition of angle bisector
3. $\angle PYX$ and $\angle PZX$ are right angles.	**3.** Definition of perpendicular
4. $\angle PYX \cong \angle PZX$	**4.** Right angles are congruent.
5. $\overline{PX} \cong \overline{PX}$	**5.** Reflexive Property
6. $\triangle PYX \cong \triangle PZX$	**6.** AAS
7. $\overline{XY} \cong \overline{XZ}$	**7.** CPCTC

CHECK Your Progress

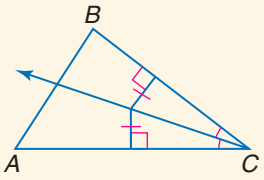
1. PROOF Write a paragraph proof of Theorem 5.1.

In Example 1, XY and XZ are lengths representing the distance from X to each side of $\angle QPR$. So, Example 1 is a proof of Theorem 5.4.

THEOREMS — *Points on Angle Bisectors*

5.4 Any point on the angle bisector is equidistant from the sides of the angle.

5.5 Any point equidistant from the sides of an angle lies on the angle bisector.



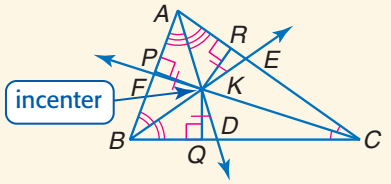
You will prove Theorem 5.5 in Exercise 24.

As with perpendicular bisectors, there are three angle bisectors in any triangle. The angle bisectors of a triangle are concurrent, and their point of concurrency is called the **incenter** of a triangle.

THEOREM 5.6 — *Incenter Theorem*

The incenter of a triangle is equidistant from each side of the triangle.

Example: If K is the incenter of $\triangle ABC$, then $KP = KQ = KR$.



You will prove Theorem 5.6 in Exercise 25.

Study Tip

Medians as Bisectors

Because the median contains the midpoint, it is also a bisector of the side of the triangle.

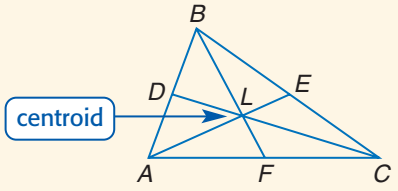
Medians and Altitudes A **median** is a segment whose endpoints are a vertex of a triangle and the midpoint of the side opposite the vertex. Every triangle has three medians.

The medians of a triangle also intersect at a common point. The point of concurrency for the medians of a triangle is called a **centroid**. The centroid is the point of balance for any triangle.

THEOREM 5.7 — *Centroid Theorem*

The centroid of a triangle is located two thirds of the distance from a vertex to the midpoint of the side opposite the vertex on a median.

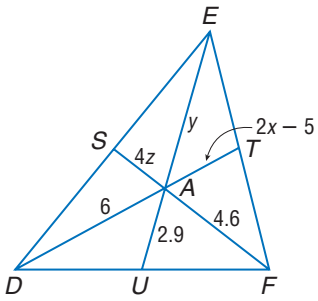
Example: If L is the centroid of $\triangle ABC$, $AL = \frac{2}{3}AE$, $BL = \frac{2}{3}BF$, and $CL = \frac{2}{3}CD$.



You can use the theorems about special segments of triangles to solve problems involving measures in triangles.

EXAMPLE Segment Measures

2 **ALGEBRA** Points S, T, and U are the midpoints of $\overline{DE}$, $\overline{EF}$, and $\overline{DF}$, respectively. Find x, y, and z.



- Find x.

$DT = DA + AT$	Segment Addition Postulate
$= 6 + (2x - 5)$	Substitution
$= 2x + 1$	Simplify.
$DA = \frac{2}{3}DT$	Centroid Theorem
$6 = \frac{2}{3}[2x + 1]$	$DA = 6$, $DT = 2x + 1$
$18 = 4x + 2$	Multiply each side by 3 and simplify.
$16 = 4x$	Subtract 2 from each side.
$4 = x$	Divide each side by 4.

- Find y.

$EA = \frac{2}{3}EU$	Centroid Theorem
$y = \frac{2}{3}(y + 2.9)$	$EA = y$, $EU = y + 2.9$
$3y = 2y + 5.8$	Multiply each side by 3 and simplify.
$y = 5.8$	Subtract $2y$ from each side.

- Find z.

$FA = \frac{2}{3}FS$	Centroid Theorem
$4.6 = \frac{2}{3}(4.6 + 4z)$	$FA = 4.6$, $FS = 4.6 + 4z$
$13.8 = 9.2 + 8z$	Multiply each side by 3 and simplify.
$4.6 = 8z$	Subtract 9.2 from each side.
$0.575 = z$	Divide each side by 8.

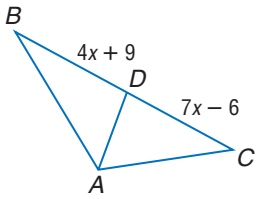
Study Tip

Eliminating Fractions

You could also multiply the equation $DA = \frac{2}{3}DT$ by 3 to eliminate the denominator.

CHECK Your Progress

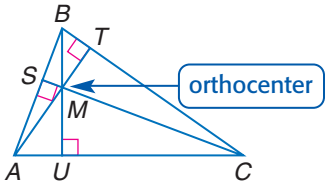
2. **ALGEBRA** Find x if $\overline{AD}$ is a median of $\triangle ABC$.



Cross-Curricular Project

Math nline Finding the orthocenter can be used to help you construct your own nine-point circle. Visit ca.geometryonline.com to continue work on your project.

An **altitude** of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. Every triangle has three altitudes. The intersection point of the altitudes of a triangle is called the **orthocenter**.



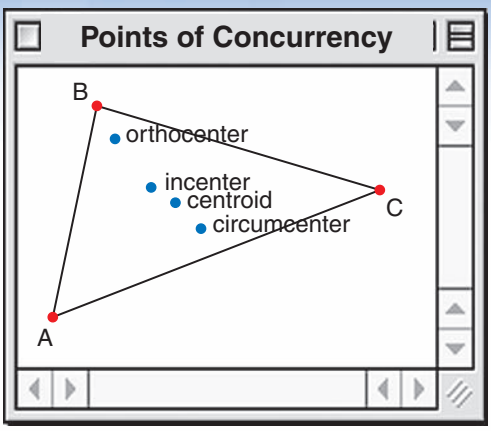
If the vertices of a triangle are located on a coordinate plane, you can use a system of equations to find the coordinates of the orthocenter.

GEOMETRY SOFTWARE LAB

Points of Concurrency

CONSTRUCT A FIGURE

- Use The Geometer's Sketchpad to construct acute scalene $\triangle ABC$.
- Construct and label the circumcenter, incenter, centroid, and orthocenter of $\triangle ABC$.

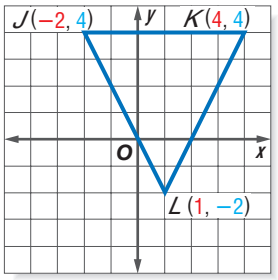


ANALYZE THE FIGURE

1. Drag the vertices of $\triangle ABC$ such that $\triangle ABC$ is a right triangle. Describe the position of each of the points of concurrency.
2. Drag a vertex of $\triangle ABC$ such that $\triangle ABC$ is an obtuse scalene triangle. Describe the position of each of the points of concurrency.
3. Explain your findings.

EXAMPLE 3 Use a System of Equations to Find a Point

COORDINATE GEOMETRY **The vertices of $\triangle JKL$ are $J(-2, 4)$, $K(4, 4)$, and $L(1, -2)$. Find the coordinates of the orthocenter of $\triangle JKL$.**



Find an equation of the altitude from J to $\overline{KL}$. The slope of $\overline{KL}$ is 2, so the slope of the altitude is $-\frac{1}{2}$.

$(y - y_1) = m(x - x_1)$ Point-slope form

$(y - 4) = -\frac{1}{2}(x - (-2))$ $(x_1, y_1) = (-2, 4)$

$y - 4 = -\frac{1}{2}x - 1$ Simplify.

$y = -\frac{1}{2}x + 3$ Add 4 to each side.

Find an equation of the altitude from K to $\overline{JL}$. The slope of $\overline{JL}$ is -2, so the slope of the altitude is $\frac{1}{2}$.

$(y - y_1) = m(x - x_1)$ Point-slope form

$y - 4 = \frac{1}{2}(x - 4)$ $(x_1, y_1) = (4, 4)$

$y - 4 = \frac{1}{2}x - 2$ Simplify.

$y = \frac{1}{2}x + 2$ Add 4 to each side.

Solve a system of equations to find the point of intersection of the altitudes.

(continued on the next page)

Study Tip

Graphing Calculator

Once you have two equations, you can graph the two lines and use the Intersect option on the Calc menu to determine where the two lines meet.

Study Tip

Simultaneous Equations

Systems of equations are also known as *simultaneous equations*, because a solution consists of values for the variables that satisfy all of the equations at the same time, or *simultaneously*.

Add to eliminate x.

$y = -\frac{1}{2}x + 3$ Equation of altitude from J

$(+)\ y = \frac{1}{2}x + 2$ Equation of altitude from K

$2y = 5$ Add.

$y = \frac{5}{2}$ or $2\frac{1}{2}$ Divide each side by 2.

Then replace y with $\frac{5}{2}$ in either equation to find x.

$y = \frac{1}{2}x + 2$

$\frac{5}{2} = \frac{1}{2}x + 2$ $y = \frac{5}{2}$

$\frac{1}{2} = \frac{1}{2}x$ Subtract 2 from each side.

$1 = x$ Divide each side by $\frac{1}{2}$.

The coordinates of the orthocenter of $\triangle JKL$ are $\left(1, 2\frac{1}{2}\right)$. To check reasonableness, draw the altitudes of each side of the triangle on the coordinate grid. The intersection is the orthocenter.

CHECK Your Progress

3. Find the circumcenter of $\triangle JKL$.

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You can also use systems of equations to find the coordinates of the circumcenter and the centroid of a triangle graphed on a coordinate plane.

CONCEPT SUMMARY *Special Segments in Triangles*

Name	Type	Point of Concurrency
perpendicular bisector	line, segment, or ray	circumcenter
angle bisector	line, segment, or ray	incenter
median	segment	centroid
altitude	segment	orthocenter

CHECK Your Understanding

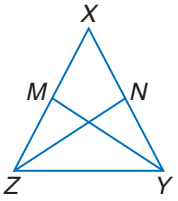
Example 1 (p. 270)

1. PROOF Write a two-column proof.

Given: $\overline{XY} \cong \overline{XZ}$

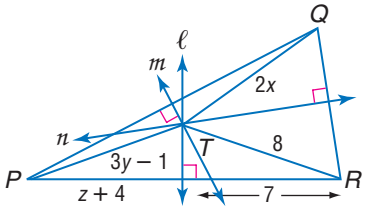
$\overline{YM}$ and $\overline{ZN}$ are medians.

Prove: $\overline{YM} \cong \overline{ZN}$



Example 2 (p. 272)

2. ALGEBRA Lines ℓ, m, and n are perpendicular bisectors of $\triangle PQR$ and meet at T. If $TQ = 2x$, $PT = 3y - 1$, and $TR = 8$, find x, y, and z.



Example 3 (pp. 273–274)

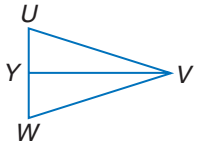
3. COORDINATE GEOMETRY The vertices of $\triangle ABC$ are $A(-3, 3)$, $B(3, 2)$, and $C(1, -4)$. Find the coordinates of the circumcenter.

Exercises

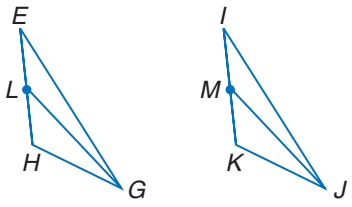
HOMEWORK HELP	
For Exercises	See Examples
4–5, 23–25	1
6–15	2
16–22	3

PROOF Write a two-column proof.

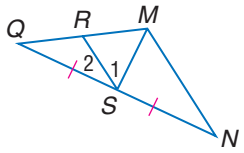
4. Given: $\triangle UVW$ is isosceles with vertex angle UVW. $\overline{YV}$ is the bisector of $\angle UVW$.
Prove: $\overline{YV}$ is a median.



5. Given: $\overline{GL}$ is a median of $\triangle EGH$. $\overline{JM}$ is a median of $\triangle IJK$. $\triangle EGH \cong \triangle IJK$
Prove: $\overline{GL} \cong \overline{JM}$



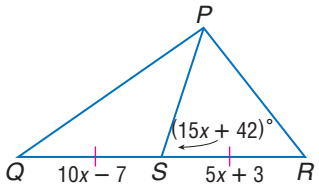
For Exercises 6 and 7, refer to $\triangle MNQ$ at the right.



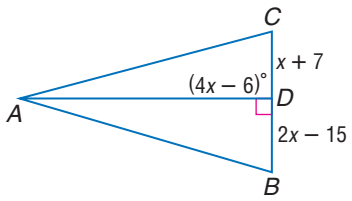
6. ALGEBRA Find x and $m\angle 2$ if $\overline{MS}$ is an altitude of $\triangle MNQ$, $m\angle 1 = 3x + 11$, and $m\angle 2 = 7x + 9$.

7. ALGEBRA If $\overline{MS}$ is a median of $\triangle MNQ$, $QS = 3a - 14$, $SN = 2a + 1$, and $m\angle MSQ = 7a + 1$, find the value of a. Is $\overline{MS}$ also an altitude of $\triangle MNQ$? Explain.

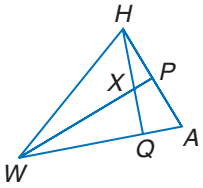
8. ALGEBRA Find x if $\overline{PS}$ is a median of $\triangle PQR$.



9. ALGEBRA Find x if $\overline{AD}$ is an altitude of $\triangle ABC$.



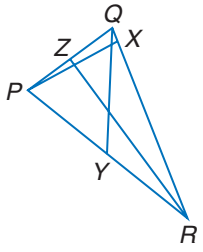
ALGEBRA For Exercises 10 and 11, refer to $\triangle WHA$ at the right.



10. If $\overline{WP}$ is a median and an angle bisector, $AP = 3y + 11$, $PH = 7y - 5$, $m\angle HWP = x + 12$, $m\angle PAW = 3x - 2$, and $m\angle HWA = 4x - 16$, find x and y. Is $\overline{WP}$ also an altitude? Explain.

11. If $\overline{WP}$ is a perpendicular bisector, $m\angle WHA = 8q + 17$, $m\angle HWP = 10 + q$, $AP = 6r + 4$, and $PH = 22 + 3r$, find r, q, and $m\angle HWP$.

ALGEBRA For Exercises 12–15, use the following information.



In $\triangle PQR$, $ZQ = 3a - 11$, $ZP = a + 5$, $PY = 2c - 1$, $YR = 4c - 11$, $m\angle PRZ = 4b - 17$, $m\angle ZRQ = 3b - 4$, $m\angle QYR = 7b + 6$, and $m\angle PXR = 2a + 10$.

12. $\overline{PX}$ is an altitude of $\triangle PQR$. Find a.

13. If $\overline{RZ}$ is an angle bisector, find $m\angle PRZ$.

14. Find PR if $\overline{QY}$ is a median.

15. If $\overleftrightarrow{QY}$ is a perpendicular bisector of $\overline{PR}$, find b.

COORDINATE GEOMETRY **The vertices of $\triangle DEF$ are $D(4, 0)$, $E(-2, 4)$, and $F(0, 6)$. Find the coordinates of the points of concurrency of $\triangle DEF$.**

16. centroid **17.** orthocenter **18.** circumcenter

COORDINATE GEOMETRY **For Exercises 19–22, use the following information.**

$R(3, 3)$, $S(-1, 6)$, and $T(1, 8)$ are the vertices of $\triangle RST$, and $\overline{RX}$ is a median.

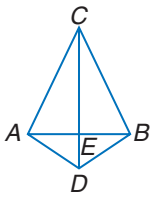
19. What are the coordinates of X?

20. Find RX.

21. Determine the slope of $\overleftrightarrow{RX}$. Then find the equation of the line.

22. Is $\overline{RX}$ an altitude of $\triangle RST$? Explain.

PROOF **Write a two-column proof for each theorem.**



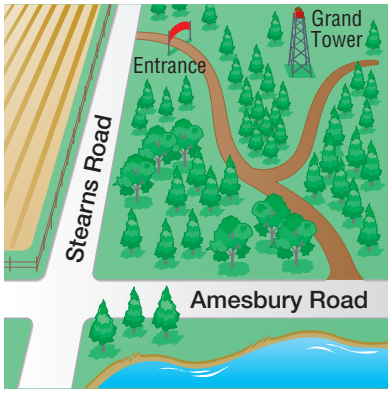
23. Theorem 5.2
Given: $\overline{CA} \cong \overline{CB}$, $\overline{AD} \cong \overline{BD}$
Prove: C and D are on the perpendicular bisector of $\overline{AB}$.

24. Theorem 5.5

25. Theorem 5.6

26. **ORIENTEERING** Orienteering is a competitive sport, originating in Sweden, that tests the skills of map reading and cross-country running. Competitors race through an unknown area to find various checkpoints using only a compass and topographical map. On an amateur course, clues are given to locate the first flag.

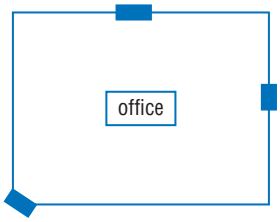
- The flag is as far from the Grand Tower as it is from the park entrance.
- If you run straight from Stearns Road to the flag or from Amesbury Road to the flag, you would run the same distance. Describe how to find the first flag.



Real-World Link
The U.S. Orienteering Federation has 71 member clubs, including the following clubs in California: Bay Area Orienteering Club in San Francisco, Gold Country Orienteer in Rocklin, the Los Angeles Orienteering Club, and San Diego Orienteering.

Source: us.orienteering.org

27. **ARCHITECTURE** An architect is designing a high school building. Describe how to position the central office so it is equidistant from each of the three entrances to the school.



STATISTICS **For Exercises 28–31, use the following information.**

The *mean* of a set of data is an average value of the data. Suppose $\triangle ABC$ has vertices $A(16, 8)$, $B(2, 4)$, and $C(-6, 12)$.

28. Find the mean of the x-coordinates of the vertices.

29. Find the mean of the y-coordinates of the vertices.

30. Graph $\triangle ABC$ and its medians.

31. **Make a conjecture** about the centroid and the means of the coordinates of the vertices.

State whether each sentence is *always*, *sometimes*, or *never* true. Justify your reasoning.

32. The three medians of a triangle intersect at a point inside the triangle.

33. The three altitudes of a triangle intersect at a vertex of the triangle.

34. The three angle bisectors of a triangle intersect at a point in the exterior of the triangle.

35. The three perpendicular bisectors of a triangle intersect at a point in the exterior of the triangle.

EXTRA PRACTICE
See pages 809, 832.

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H.O.T. Problems

36. REASONING Compare and contrast a perpendicular bisector and a median of a triangle.

37. REASONING Find a counterexample to the statement *An altitude and an angle bisector of a triangle are never the same segment*.

38. OPEN ENDED Draw a triangle in which the circumcenter lies outside the triangle.

39. Which One Doesn't Belong? Identify the term that does not belong with the other three. Explain your reasoning.

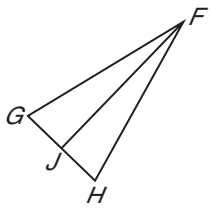
orthocenter	point of concurrency	altitude	circumcenter

40. CHALLENGE Draw any $\triangle XYZ$ with median $\overline{XN}$ and altitude $\overline{XO}$. Recall that the area of a triangle is one-half the product of the measures of the base and the altitude. What conclusion can you make about the relationship between the areas of $\triangle XYN$ and $\triangle XZN$?

41. *Writing in Math* Explain how to balance a paper triangle on a pencil point. Is it possible for the incenter of a triangle to be the center of gravity?

STANDARDS PRACTICE

42. In the figure below, $\overline{GJ} \cong \overline{HJ}$.



Which statement about $\overline{FJ}$ *must* be true?

A $\overline{FJ}$ is an angle bisector of $\triangle FGH$.

B $\overline{FJ}$ is a perpendicular bisector of $\triangle FGH$.

C $\overline{FJ}$ is a median of $\triangle FGH$.

D $\overline{FJ}$ is an altitude of $\triangle FGH$.

43. REVIEW An object that is projected straight upward with initial velocity v meters per second travels an estimated distance of $s = -vt + 10t^2$, where t = time in seconds. If Sherise is standing at the edge of a balcony 54 meters above the ground and throws a ball straight up with an initial velocity of 12 meters per second, after how many seconds will it hit the ground?

F 3 seconds

G 4 seconds

H 6 seconds

J 9 seconds

Spiral Review

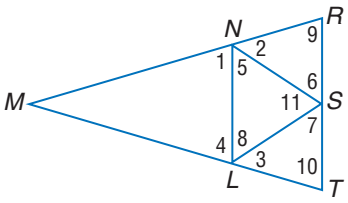
Position and label each triangle on the coordinate plane. (Lesson 4-7)

44. equilateral $\triangle ABC$ with base $\overline{AB}$ that is n units long

45. isosceles $\triangle DEF$ with congruent sides $2a$ units long and base a units long

46. right $\triangle GHI$ with hypotenuse $\overline{GI}$, HI is three times GH, and $\overline{GH}$ is x units long

For Exercises 47–50, refer to the figure at the right. (Lesson 4-6)



47. If $\angle 9 \cong \angle 10$, name two congruent segments.

48. If $\overline{NL} \cong \overline{SL}$, name two congruent angles.

49. If $\overline{LT} \cong \overline{LS}$, name two congruent angles.

50. If $\angle 1 \cong \angle 4$, name two congruent segments.

51. INTERIOR DESIGN Stacey is installing a curtain rod on the wall above the window. To ensure that the rod is parallel to the ceiling, she measures and marks 6 inches below the ceiling in several places. If she installs the rod at these markings centered over the window, how does she know the curtain rod will be parallel to the ceiling? (Lesson 3-6)

Determine the slope of the line that contains the given points. (Lesson 3-3)

52. $A(0, 6)$, $B(4, 0)$ **53.** $G(8, 1)$, $H(8, -6)$ **54.** $E(6, 3)$, $F(-6, 3)$

55. Copy and complete the truth table. (Lesson 2-2)

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

Determine whether each conjecture is *true* or *false*. Give a counterexample for any false conjecture. (Lesson 2-1)

56. Given: x is an integer.
Conjecture: $-x$ is negative.

57. Given: $WXYZ$ is a rectangle.
Conjecture: $WX = YZ$ and $WZ = XY$

58. $\angle L$ and $\angle M$ are complementary angles. $\angle N$ and $\angle P$ are complementary angles. If $m\angle L = y - 2$, $m\angle M = 2x + 3$, $m\angle N = 2x - y$, and $m\angle P = x - 1$, find the values of x, y, $m\angle L$, $m\angle M$, $m\angle N$, and $m\angle P$. (Lesson 1-5)

GET READY for the Next Lesson

PREREQUISITE SKILL **Replace each ● with $<$ or $>$ to make each sentence true.**

59. $\frac{3}{8}$ ● $\frac{5}{16}$ **60.** 2.7 ● $\frac{5}{3}$ **61.** -4.25 ● $-\frac{19}{4}$ **62.** $-\frac{18}{25}$ ● $-\frac{19}{27}$

READING MATH

Standard 3.0 **Students construct** and judge the validity of **a logical argument** and give counterexamples to disprove a statement. (Key)

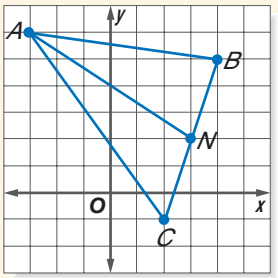
Writing Explanations

Often in mathematics, simply providing an answer is not sufficient. You must be able to show understanding by explaining your answers or justifying your reasoning.

EXAMPLE

Is $\overline{AN}$ an altitude of $\triangle ABC$? Justify your reasoning.

It is not enough to say that $\overline{AN}$ is not an altitude of $\triangle ABC$ because "it does not look like it." You must support your reasoning.



$$\text{slope of } \overline{AN} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{2 - 6}{3 - (-3)} \quad (x_1, y_1) = (-3, 6), (x_2, y_2) = (3, 2)$$

$$= -\frac{2}{3} \quad \text{Simplify.}$$

$$\text{slope of } \overline{BC} = \frac{y_2 - y_1}{x_2 - x_1}$$

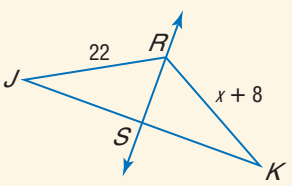
$$= \frac{-1 - 5}{2 - 4} \quad (x_1, y_1) = (4, 5), (x_2, y_2) = (2, -1)$$

$$= 3 \quad \text{Simplify.}$$

Complete Answer:
The product of the slopes of $\overline{AN}$ and $\overline{BC}$ is not -1, so the segments are not perpendicular. Therefore, $\overline{AN}$ is not an altitude of $\triangle ABC$.

Reading to Learn

1. Describe some ways that you can explain your answer or justify your reasoning in mathematics.
2. Refer to the graph of $\triangle ABC$ above. Is $\overline{AN}$ a median of $\triangle ABC$? Justify your reasoning.
3. Refer to $\triangle RKJ$ shown at the right. $\overleftrightarrow{RS}$ is a perpendicular bisector of $\overline{JK}$. What is the value of x? Explain.



4. In $\triangle XYZ$, $XY = 15$ centimeters, $YZ = 12$ centimeters, and $ZX = 23$ centimeters. List the angles from greatest to least measure. Explain your reasoning.
5. How is writing explanations and justifications useful in making decisions and critical judgments in problem situations?

5-2 Inequalities and Triangles

Main Ideas

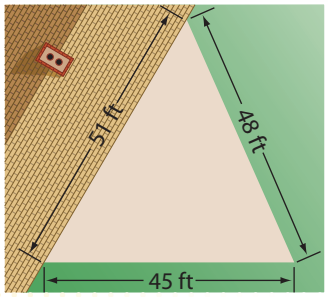
- Recognize and apply properties of inequalities to the measures of angles of a triangle.
- Recognize and apply properties of inequalities to the relationships between angles and sides of a triangle.

Standard 12.0 **Students find and use measures of** sides and of interior and **exterior angles of triangles** and polygons **to** classify figures and **solve problems.** (Key)

Standard 13.0 **Students prove relationships between angles in polygons by using properties** of complementary, supplementary, vertical, and **exterior angles.**

GET READY for the Lesson

Bryan is delivering a potted tree for a patio. The tree is to be placed in the largest corner of the patio. All Bryan has is a diagram of the triangular patio that shows the measurements. Bryan can find the *largest corner* because the measures of the angles of a triangle are related to the measures of the sides opposite them.



Angle Inequalities In algebra, you learned about the inequality relationship between two real numbers. This relationship is often used in proofs.

KEY CONCEPT *Definition of Inequality*

For any real numbers a and b, $a > b$ if and only if there is a positive number c such that $a = b + c$.

Example: If $6 = 4 + 2$, $6 > 4$ and $6 > 2$.

The table below lists several properties of inequalities you studied in algebra. These properties can be applied to the measures of angles and segments since these are real numbers.

Properties of Inequalities for Real Numbers	
	For all numbers a, b, and c
Comparison Property	$a < b$, $a = b$, or $a > b$
Transitive Property	**1.** If $a < b$ and $b < c$, then $a < c$. **2.** If $a > b$ and $b > c$, then $a > c$.
Addition and Subtraction Properties	**1.** If $a > b$, then $a + c > b + c$ and $a - c > b - c$. **2.** If $a < b$, then $a + c < b + c$ and $a - c < b - c$.
Multiplication and Division Properties	**1.** If $c > 0$ and $a < b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$. **2.** If $c > 0$ and $a > b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. **3.** If $c < 0$ and $a < b$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$. **4.** If $c < 0$ and $a > b$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.

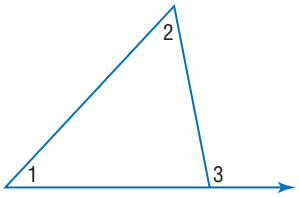
Study Tip

Symbols for Angles and Inequalities

The symbol for angle (∠) looks similar to the symbol for less than (<), especially when handwritten. Be careful to write the symbols correctly in situations where both are used.

EXAMPLE 1 Compare Angle Measures

Determine which angle has the greatest measure.



Explore Compare the measure of ∠3 to the measures of ∠1 and ∠2.

Plan Use properties and theorems of real numbers to compare the angle measures.

Solve Compare $m\angle 1$ to $m\angle 3$.
By the Exterior Angle Theorem, $m\angle 3 = m\angle 1 + m\angle 2$.
Since angle measures are positive numbers and from the definition of inequality, $m\angle 3 > m\angle 1$.

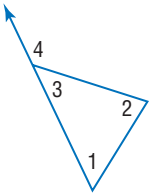
Compare $m\angle 2$ to $m\angle 3$.

Again, by the Exterior Angle Theorem, $m\angle 3 = m\angle 1 + m\angle 2$.
The definition of inequality states that if $m\angle 3 = m\angle 1 + m\angle 2$, then $m\angle 3 > m\angle 2$.

Check $m\angle 3$ is greater than $m\angle 1$ and $m\angle 2$. Therefore, ∠3 has the greatest measure.

CHECK Your Progress

1. Determine which angle has the greatest measure.



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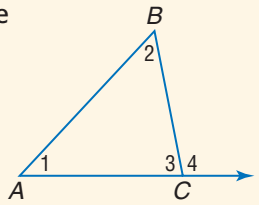
The results from Example 1 suggest that the measure of an exterior angle is always greater than either of the measures of the remote interior angles.

THEOREM 5.8 Exterior Angle Inequality

If an angle is an exterior angle of a triangle, then its measure is greater than the measure of either of its corresponding remote interior angles.

Example: $m\angle 4 > m\angle 1$

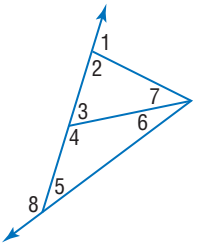
$m\angle 4 > m\angle 2$



The proof of Theorem 5.8 is in Lesson 5-3.

EXAMPLE 2 Exterior Angles

Use the Exterior Angle Inequality Theorem to list all of the angles that satisfy the stated condition.



a. measures less than $m\angle 8$

By the Exterior Angle Inequality Theorem, $m\angle 8 > m\angle 4$, $m\angle 8 > m\angle 6$, $m\angle 8 > m\angle 2$, and $m\angle 8 > m\angle 6 + m\angle 7$. Thus, the measures of ∠4, ∠6, ∠2, and ∠7 are all less than $m\angle 8$.

(continued on the next page)

b. measures greater than $m\angle 2$

By the Exterior Angle Inequality Theorem, $m\angle 8 > m\angle 2$ and $m\angle 4 > m\angle 2$. Thus, the measures of $\angle 4$ and $\angle 8$ are greater than $m\angle 2$.

CHECK Your Progress

2. measures less than $\angle 3$

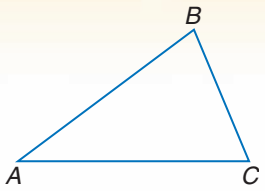
Angle-Side Relationships Recall that if two sides of a triangle are congruent, then the angles opposite those sides are congruent. In the following Geometry Activity, you will investigate the relationship between sides and angles when they are not congruent.

GEOMETRY LAB

Inequalities for Sides and Angles of Triangles

MODEL

Step 1 Draw an acute scalene triangle, and label the vertices A, B, and C.



Step 2 Measure each side of the triangle. Record the measures in a table.

Side	Measure
$\overline{BC}$	
$\overline{AC}$	
$\overline{AB}$	

Step 3 Measure each angle of the triangle. Record each measure in a table.

Angle	Measure
$\angle A$	
$\angle B$	
$\angle C$	

ANALYZE

1. Describe the measure of the angle opposite the longest side in terms of the other angles.
2. Describe the measure of the angle opposite the shortest side in terms of the other angles.
3. Repeat the activity using other triangles.

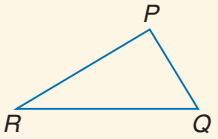
MAKE A CONJECTURE

4. What can you conclude about the relationship between the measures of sides and angles of a triangle?

The Geometry Lab suggests the following theorem.

THEOREM 5.9

If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the shorter side.



Study Tip

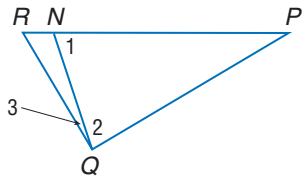
Theorem 5.9

The longest side in a triangle is opposite the largest angle in that triangle.

PROOF Theorem 5.9

Given: $\triangle PQR$
$PQ < PR$
$\overline{PN} \cong \overline{PQ}$

Prove: $m\angle R < m\angle PQR$



Proof:

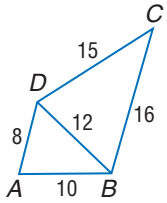
Statements	Reasons
1. $\triangle PQR$, $PQ < PR$, $\overline{PN} \cong \overline{PQ}$	**1.** Given
2. $\angle 1 \cong \angle 2$	**2.** Isosceles Triangle Theorem
3. $m\angle 1 = m\angle 2$	**3.** Definition of congruent angles
4. $m\angle R < m\angle 1$	**4.** Exterior Angle Inequality Theorem
5. $m\angle 2 + m\angle 3 = m\angle PQR$	**5.** Angle Addition Postulate
6. $m\angle 2 < m\angle PQR$	**6.** Definition of inequality
7. $m\angle 1 < m\angle PQR$	**7.** Substitution Property of Equality
8. $m\angle R < m\angle PQR$	**8.** Transitive Property of Inequality

EXAMPLE Side-Angle Relationships

3 **Determine the relationship between the measures of the given angles.**

a. $\angle ADB, \angle DBA$

The side opposite $\angle ADB$ is longer than the side opposite $\angle DBA$, so $m\angle ADB > m\angle DBA$.



b. $\angle CDA, \angle CBA$

$$m\angle DBA < m\angle ADB$$
$$m\angle CBD < m\angle CDB$$
$$m\angle DBA + m\angle CBD < m\angle ADB + m\angle CDB$$
$$m\angle CBA < m\angle CDA$$

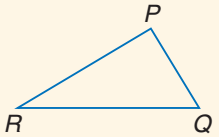
CHECK Your Progress

3. $\angle CBD, \angle CDB$

The converse of Theorem 5.9 is also true.

THEOREM 5.10

If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.



You will prove Theorem 5.10 in Lesson 5-3, Exercise 21.

Tony Freeman/PhotoEdit

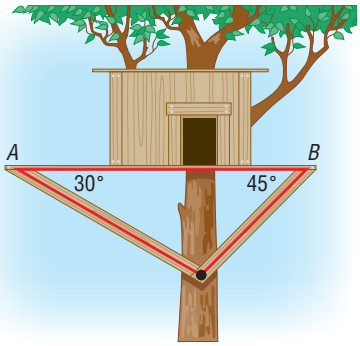
Real-World Link

The strength of the tree is the most important concern when building a treehouse. It is important to look for a tree that has thick, strong branches.

Source: www.treehouses.com

Real-World EXAMPLE Angle-Side Relationships

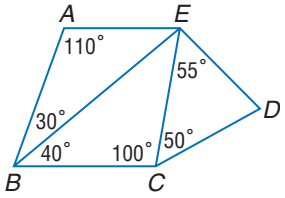
4 **TREEHOUSES** **Mrs. Sanchez is constructing the framework for part of a treehouse for her daughter. She plans to install braces at the ends of a certain floor support as shown. Which brace will be longer—the brace attached to *A* or to *B*?**



Theorem 5.10 states that if one angle of a triangle has a greater measure, then the side opposite that angle is longer than the side opposite the other angle. Therefore, the brace attached to the end marked *A* will be longer than the brace attached to the end marked *B*.

CHECK Your Progress

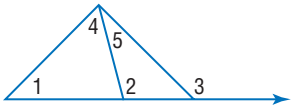
4. Determine the relationship between *BC* and *EC*.



CHECK Your Understanding

Example 1 (p. 281)

Determine which angle has the greatest measure.



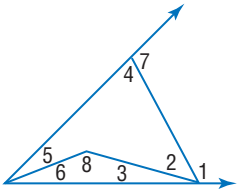
1. $\angle 1, \angle 2, \angle 4$

2. $\angle 2, \angle 3, \angle 5$

3. $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$

Example 2 (pp. 281–282)

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.



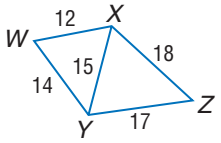
4. measures less than $m\angle 1$

5. measures greater than $m\angle 6$

6. measures less than $m\angle 7$

Example 3 (p. 283)

Determine the relationship between the measures of the given angles.



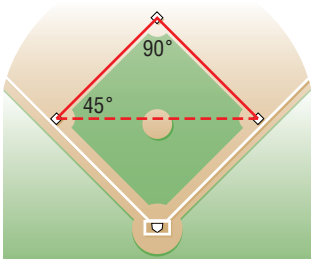
7. $\angle WXY, \angle XYW$

8. $\angle XZY, \angle XYZ$

9. $\angle WYX, \angle XWY$

Example 4 (p. 284)

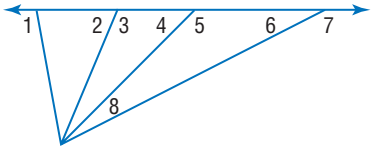
10. **BASEBALL** During a baseball game, the batter hits the ball to the third baseman and begins to run toward first base. At the same time, the runner on first base runs toward second base. If the third baseman wants to throw the ball to the nearest base, to which base should he throw? Explain.



Exercises

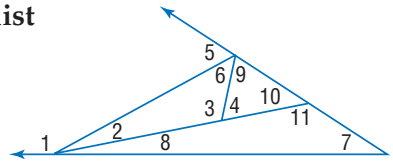
HOMEWORK HELP	
For Exercises	See Examples
11–16	1
17–20	2
21–26	3
27–32	4

Determine which angle has the greatest measure.



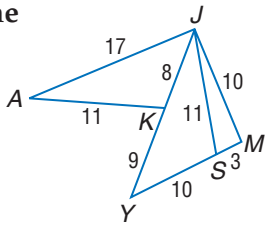
11. $\angle 1, \angle 2, \angle 4$
12. $\angle 2, \angle 4, \angle 6$
13. $\angle 3, \angle 5, \angle 7$
14. $\angle 1, \angle 2, \angle 6$
15. $\angle 5, \angle 7, \angle 8$
16. $\angle 2, \angle 6, \angle 8$

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition.



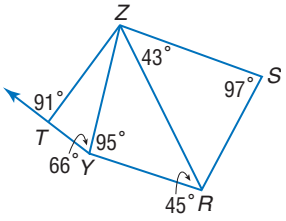
17. measures less than $m\angle 5$
18. measures greater than $m\angle 6$
19. measures greater than $m\angle 10$
20. measures less than $m\angle 11$

Determine the relationship between the measures of the given angles.



21. $\angle KAJ, \angle AJK$
22. $\angle MJY, \angle JYM$
23. $\angle SMJ, \angle MJS$
24. $\angle AKJ, \angle JAK$
25. $\angle MYJ, \angle JMY$
26. $\angle JSY, \angle JYS$

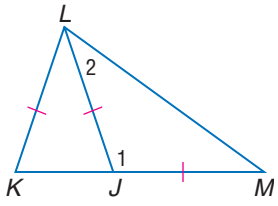
Determine the relationship between the lengths of the given sides.



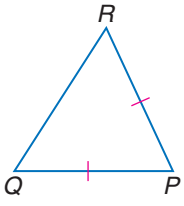
27. $\overline{ZY}, \overline{YR}$
28. $\overline{SR}, \overline{ZS}$
29. $\overline{RZ}, \overline{SR}$
30. $\overline{ZY}, \overline{RZ}$
31. $\overline{TY}, \overline{ZY}$
32. $\overline{TY}, \overline{ZT}$

PROOF **Write a two-column proof.**

33. **Given:** $\overline{JM} \cong \overline{JL}$
$\overline{JL} \cong \overline{KL}$
Prove: $m\angle 1 > m\angle 2$



34. **Given:** $\overline{PR} \cong \overline{PQ}$
$QR > QP$
Prove: $m\angle P > m\angle Q$



Real-World Link
One sixth of adult Americans have never flown in a commercial aircraft.

Source: U.S. Bureau of Transportation Statistics

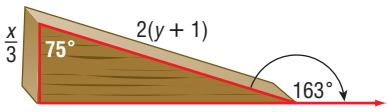
35. **TRAVEL** A plane travels from Chicago to Atlanta, on to Austin, and then completes the trip directly back to Chicago as shown in the diagram. Name the legs of the trip in order from longest to shortest.

Jeff Greenberg/PhotoEdit

36. COORDINATE GEOMETRY Triangle KLM has vertices $K(3, 2)$, $L(-1, 5)$, and $M(-3, -7)$. List the angles in order from the least to the greatest measure.

37. If $AB > AC > BC$ in $\triangle ABC$ and $\overline{AM}$, $\overline{BN}$, and $\overline{CO}$ are the medians of the triangle, list AM, BN, and CO in order from least to greatest.

38. SKATEBOARDING The wedge at the right represents a skateboard ramp. The values of x and y are in inches. Write an inequality relating x and y. Then solve the inequality for y in terms of x.



ALGEBRA Find the value of n. List the sides of $\triangle PQR$ in order from shortest to longest for the given angle measures.

39. $m\angle P = 9n + 29$, $m\angle Q = 93 - 5n$, $m\angle R = 10n + 2$

40. $m\angle P = 12n - 9$, $m\angle Q = 62 - 3n$, $m\angle R = 16n + 2$

41. $m\angle P = 9n - 4$, $m\angle Q = 4n - 16$, $m\angle R = 68 - 2n$

42. $m\angle P = 3n + 20$, $m\angle Q = 2n + 37$, $m\angle R = 4n + 15$

43. $m\angle P = 4n + 61$, $m\angle Q = 67 - 3n$, $m\angle R = n + 74$

EXTRA PRACTICE
See pages 809, 832.
Math Online
Self-Check Quiz at
ca.geometryonline.com

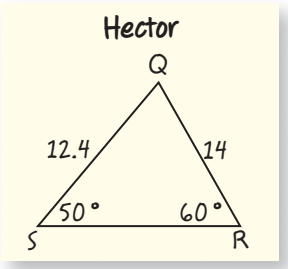
44. PROOF Write a paragraph proof for the following statement.
If a triangle is not isosceles, then the measure of the median to any side of the triangle is greater than the measure of the altitude to that side.

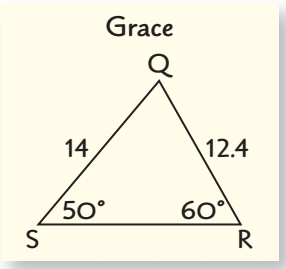
H.O.T. Problems

45. REASONING Is the following statement *always*, *sometimes*, or *never* true? Justify your answer.
In $\triangle JKL$ with right angle J, if $m\angle J$ is twice $m\angle K$, then the side opposite $\angle J$ is twice the length of the side opposite $\angle K$.

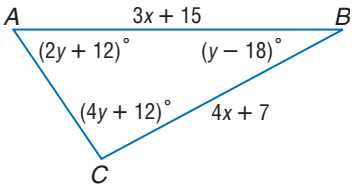
46. OPEN ENDED Draw $\triangle ABC$ such that $m\angle A > m\angle B > m\angle C$. Do not measure the angles. Explain how you know the greatest and least angle measures.

47. FIND THE ERROR Hector and Grace each labeled $\triangle QRS$. Who is correct? Explain.





48. CHALLENGE Write and solve an inequality for x.



49. ***Writing in Math*** Refer to the diagram on page 280. How can you tell which corner is largest? Include the name of the theorem or postulate that lets you determine the comparison of the angle measures and which angles in the diagram are the largest.

STANDARDS PRACTICE

50. Two angles of a triangle have measures 45° and 92°. What type of triangle is it?

A obtuse scalene

B obtuse isosceles

C acute scalene

D acute isosceles

51. What is the x-intercept of the graph of $4x - 6y = 12$?

F -3 **H** 2

G -2 **J** 3

52. REVIEW The chart below describes the speed of four students folding letters to be mailed to local businesses.

Student	Description
Neiva	Folds 1 page every 3 seconds
Sarah	Folds 2 pages every 5 seconds
Quin	Folds 100 pages per minute
Deron	Folds 180 pages in 2 minutes

Which student is the *fastest*?

A Sarah **C** Neiva

B Quin **D** Deron

Spiral Review

ALGEBRA For Exercises 53–55, use the following information. (Lesson 5-1)

Two vertices of $\triangle ABC$ are $A(3, 8)$ and $B(9, 12)$. $\overline{AD}$ is a median with D at $(12, 3)$.

53. What are the coordinates of C?

54. Is $\overline{AD}$ an altitude of $\angle ABC$? Explain.

55. The graph of point E is at $(6, 6)$. $\overline{EF}$ intersects $\overline{BD}$ at F. If F is at $\left(10\frac{1}{2}, 7\frac{1}{2}\right)$, is $\overline{EF}$ a perpendicular bisector of $\overline{BD}$? Explain.

56. AMUSEMENT PARK Miguel and his friends are at the Ferris wheel. They head 50 feet east to the snack hut. Then Miguel and a friend head north 25 feet to wait in line for a roller coaster ride. The rest of their group continues walking east 50 feet to the water park. Write a coordinate proof to prove that the Ferris wheel, the end of the line for the roller coaster, and the water park form an isosceles triangle. (Lesson 4-7)

Name the corresponding congruent angles and sides for each pair of congruent triangles. (Lesson 4-3)

57. $\triangle TUV \cong \triangle XYZ$ **58.** $\triangle CDG \cong \triangle RSW$ **59.** $\triangle BCF \cong \triangle DGH$

60. Find the value of x so that the line containing points at $(x, 2)$ and $(-4, 5)$ is perpendicular to the line containing points at $(4, 8)$ and $(2, -1)$. (Lesson 3-3)

GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether each equation is *true* or *false* if $a = 2$, $b = 5$, and $c = 6$.

61. $2ab = 20$ **62.** $c(b - a) = 15$ **63.** $a + c > a + b$

5-3 Indirect Proof

Main Ideas

- Use indirect proof with algebra.
- Use indirect proof with geometry.

Standard 2.0 **Students write geometric proofs, including proofs by contradiction.** (Key)

New Vocabulary

indirect reasoning
indirect proof
proof by contradiction

GET READY for the Lesson

In *The Adventure of the Blanched Soldier*, Sherlock Holmes describes his detective technique, stating, "That process starts upon the supposition that when you have eliminated all which is impossible, then whatever remains, . . . must be the truth." The method Sherlock Holmes uses is an example of *indirect reasoning*.

Indirect Proof with Algebra The proofs you have written so far use direct reasoning, in which you start with a true hypothesis and prove that the conclusion is true. When using **indirect reasoning**, you assume that the conclusion is false and then show that this assumption leads to a contradiction of the hypothesis, or some other accepted fact, such as a definition, postulate, theorem, or corollary. Since all other steps in the proof are logically correct, the assumption has been proven false, so the original conclusion must be true. A proof of this type is called an **indirect proof** or a **proof by contradiction**. The following steps summarize the process of an indirect proof.

Study Tip

Truth Value of a Statement

Recall that a statement must be either true or false. To review **truth values**, see Lesson 2-2.

KEY CONCEPT *Writing an Indirect Proof*

1. Assume that the conclusion is false.
2. Show that this assumption leads to a contradiction of the hypothesis, or some other fact, such as a definition, postulate, theorem, or corollary.
3. Point out that because the false conclusion leads to an incorrect statement, the original conclusion must be true.

EXAMPLE **State Assumptions**

1 **State the assumption you would make to start an indirect proof of each statement.**

a. $AB \neq MN$

$AB = MN$

b. **$\triangle PQR$ is an isosceles triangle.**

$\triangle PQR$ is not an isosceles triangle.

c. **If 9 is a factor of n, then 3 is a factor of n.**

The conclusion of the conditional statement is *3 is a factor of n*. The negation of the conclusion is *3 is not a factor of n*.

CHECK Your Progress

1A. $x < 4$

1B. $\angle 3$ is an obtuse angle.

Indirect proofs can be used to prove algebraic concepts.

EXAMPLE 2 Algebraic Proof

Given: $2x - 3 > 7$

Prove: $x > 5$

Indirect Proof:

Step 1 Assume that $x \leq 5$. That is, assume that $x < 5$ or $x = 5$.

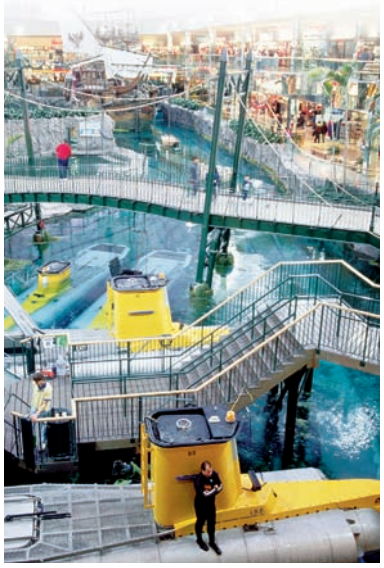
Step 2 Make a table with several possibilities for x given that $x < 5$ or $x = 5$. This is a contradiction because when $x < 5$ or $x = 5$, $2x - 3 \leq 7$.

x	$2x - 3$
1	−1
2	1
3	3
4	5
5	7

Step 3 In both cases, the assumption leads to the contradiction of a known fact. Therefore, the assumption that $x \leq 5$ must be false, which means that $x > 5$ must be true.

CHECK Your Progress

2. If $7x < 56$, then $x < 8$.



James Marshall/CORBIS

Indirect reasoning and proof can be used in everyday situations.

Real-World EXAMPLE 3

SHOPPING **Lawanda bought two skirts for just over $60, before tax. A few weeks later, her friend Tiffany asked her how much each skirt cost. Lawanda could not remember the individual prices. Use indirect reasoning to show that at least one of the skirts cost more than $30.**

Given: The two skirts cost more than $60.

Prove: At least one of the skirts cost more than $30.
That is, if $x + y > 60$, then either $x > 30$ or $y > 30$.

Indirect Proof:

Step 1 Assume that neither skirt costs more than $30. That is, $x \leq 30$ and $y \leq 30$.

Step 2 If $x \leq 30$ and $y \leq 30$, then $x + y \leq 60$. This is a contradiction because we know that the two skirts cost more than $60.

Step 3 The assumption leads to the contradiction of a known fact. Therefore, the assumption that $x \leq 30$ and $y \leq 30$ must be false. Thus, at least one of the skirts had to have cost more than $30.

CHECK Your Progress

3. Ben traveled over 360 miles and made one stop. Use indirect reasoning to prove that he traveled more than 180 miles on one part of his trip.

Online Personal Tutor at ca.geometryonline.com

Real-World Link

The West Edmonton Mall in Edmonton, Alberta, Canada, is the world's largest entertainment and shopping center, with an area of 5.3 million square feet. The mall houses an amusement park, water park, ice rink, and aquarium, along with over 800 stores and services.

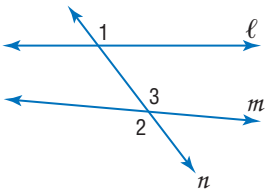
Source: westedmall.com

Indirect Proof with Geometry Indirect reasoning can be used to prove statements in geometry.

EXAMPLE Geometry Proof

4 **Given:** $\ell \nparallel m$

Prove: $\angle 1 \ncong \angle 3$



Indirect Proof:

Step 1 Assume that $\angle 1 \cong \angle 3$.

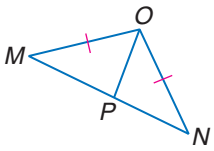
Step 2 $\angle 1$ and $\angle 3$ are corresponding angles. If two lines are cut by a transversal so that corresponding angles are congruent, the lines are parallel. This means that $\ell \parallel m$. However, this contradicts the given statement.

Step 3 Since the assumption leads to a contradiction, the assumption must be false. Therefore, $\angle 1 \ncong \angle 3$.

CHECK Your Progress

4. Given: $\overline{MO} \cong \overline{ON}$, $\overline{MP} \ncong \overline{NP}$

Prove: $\angle MOP \ncong \angle NOP$

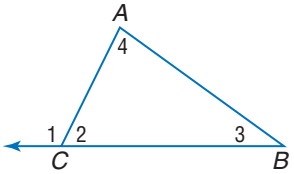


Indirect proofs can also be used to prove theorems.

Proof Exterior Angle Inequality Theorem

Given: $\angle 1$ is an exterior angle of $\triangle ABC$.

Prove: $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$



Indirect Proof:

Step 1 Make the assumption that $m\angle 1 \ngtr m\angle 3$ or $m\angle 1 \ngtr m\angle 4$. In other words, $m\angle 1 \leq m\angle 3$ or $m\angle 1 \leq m\angle 4$.

Step 2 You only need to show that the assumption $m\angle 1 \leq m\angle 3$ leads to a contradiction as the argument for $m\angle 1 \leq m\angle 4$ follows the same reasoning.

$m\angle 1 \leq m\angle 3$ means that either $m\angle 1 = m\angle 3$ or $m\angle 1 < m\angle 3$.

Case 1: $m\angle 1 = m\angle 3$

$m\angle 1 = m\angle 3 + m\angle 4$ Exterior Angle Theorem

$m\angle 3 = m\angle 3 + m\angle 4$ Substitution

$0 = m\angle 4$ Subtract $m\angle 3$ from each side.

This contradicts the fact that the measure of an angle is greater than 0, so $m\angle 1 \neq m\angle 3$.

Case 2: $m\angle 1 < m\angle 3$

By the Exterior Angle Theorem, $m\angle 1 = m\angle 3 + m\angle 4$. Since angle measures are positive, the definition of inequality implies $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$. This contradicts the assumption.

Step 3 In both cases, the assumption leads to the contradiction of a theorem or definition. Therefore, the assumption that $m\angle 1 > m\angle 3$ and $m\angle 1 > m\angle 4$ must be true.

Study Tip

Inequalities

Notice that the opposite of $m\angle 1 > m\angle 3$ is $m\angle 1 \leq m\angle 3$, not $m\angle 1 < m\angle 3$.

Concepts in Motion

Interactive Lab

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CHECK Your Understanding

Example 1 (p. 288)

Write the assumption you would make to start an indirect proof of each statement.

1. If $5x < 25$, then $x < 5$.
2. Two lines that are cut by a transversal so that alternate interior angles are congruent are parallel.
3. If the alternate interior angles formed by two lines and a transversal are congruent, the lines are parallel.

Example 2 (p. 289)

PROOF Write an indirect proof.

4. **Given:** $a > 0$
   Prove: $\frac{1}{a} > 0$
5. **Given:** n is odd.
   Prove: n^2 is odd.

Example 3 (p. 289)

6. **BICYCLING** The Tour de France bicycle race takes place over several weeks in various stages throughout France. During the first two stages of the 2005 Tour de France, riders raced for just over 200 kilometers. Prove that at least one of the stages was longer than 100 kilometers.

Example 4 (p. 290)

7. **PROOF** Use an indirect proof to show that the hypotenuse of a right triangle is the longest side.

Exercises

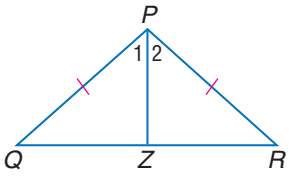
HOMEWORK HELP	
For Exercises	See Examples
8–13	1
14, 15	2
16–21	3, 4

Write the assumption you would make to start an indirect proof of each statement.

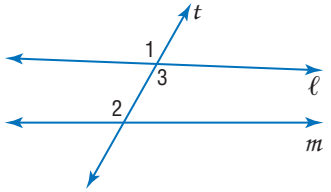
8. $\overline{PQ} \cong \overline{ST}$
9. If $3x > 12$, then $x > 4$.
10. If a rational number is any number that can be expressed as $\frac{a}{b}$, where a and b are integers and $b \neq 0$, 6 is a rational number.
11. A median of an isosceles triangle is also an altitude.
12. Points P, Q, and R are collinear.
13. The angle bisector of the vertex angle of an isosceles triangle is also an altitude of the triangle.

PROOF For Exercises 14–19, write an indirect proof.

14. **Given:** $\frac{1}{a} < 0$
    Prove: a is negative.
15. **Given:** n^2 is even.
    Prove: n^2 is divisible by 4.
16. If $a > 0$, $b > 0$, and $a > b$, then $\frac{a}{b} > 1$.
17. If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent.
18. **Given:** $\overline{PQ} \cong \overline{PR}$
    $\angle 1 \ncong \angle 2$
    Prove: $\overline{PZ}$ is not a median of $\triangle PQR$.



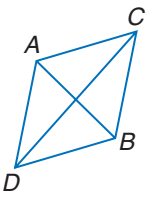
19. **Given:** $m\angle 2 \neq m\angle 1$
    Prove: $\ell \nparallel m$



PROOF **For Exercises 20 and 21, write an indirect proof.**

20. **Given:** $\triangle ABC$ and $\triangle ABD$ are equilateral. $\triangle ACD$ is not equilateral.

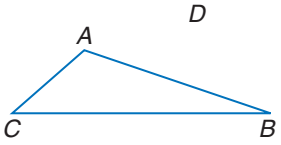
Prove: $\triangle BCD$ is not equilateral.



21. Theorem 5.10

Given: $m\angle A > m\angle ABC$

Prove: $BC > AC$



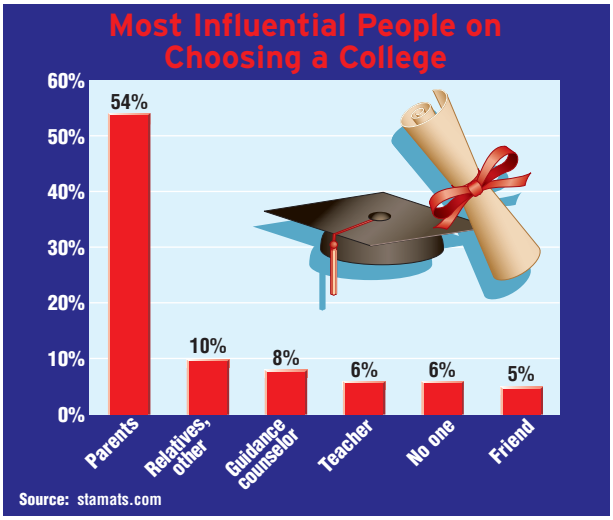
22. **BASKETBALL** Ramon scored 85 points for his high school basketball team during the last six games. Prove that his average points per game were less than 15.

COLLEGE **For Exercises 23–25, refer to the graphic.**

23. Prove the following statement. *The majority of college-bound seniors stated that their parents were the most influential people in their choice of a college.*

24. If 1500 seniors were polled for this survey, verify that 75 said a friend influenced their decision most.

25. Were more seniors most influenced by their guidance counselors or by their teachers and friends? Explain.



26. **LAW** During the opening arguments of a trial, a defense attorney stated, "My client is innocent. The police report states that the crime was committed on November 6 at approximately 10:15 A.M. in San Diego. I can prove that my client was on vacation in Chicago with his family at this time. A verdict of not guilty is the only possible verdict." Explain whether this is an example of indirect reasoning.

27. **GAMES** Use indirect reasoning and a chart to solve this problem. A computer game involves a knight on a quest for treasure. At the end of the journey, the knight approaches two doors. A sign on the door on the right reads *Behind this door is a treasure chest and behind the other door is a ferocious dragon.* The door on the left has a sign that reads *One of these doors leads to a treasure chest and the other leads to a ferocious dragon.* A servant tells the knight that one of the signs is true and the other is false. Which door should the knight choose? Explain your reasoning.

EXTRA PRACTICE
See pages 810, 832.
MathOnline
Self-Check Quiz at
ca.geometryonline.com

H.O.T. Problems

28. **REASONING** Compare and contrast indirect proof and direct proof.

29. **OPEN ENDED** State a conjecture. Then write an indirect proof to prove your conjecture.

30. CHALLENGE Recall that a rational number is any number that can be expressed in the form $\frac{a}{b}$, where a and b are integers with no common factors and $b \neq 0$, or as a terminating or repeating decimal. Use indirect reasoning to prove that $\sqrt{2}$ is not a rational number.

31. *Writing in Math* Refer to the information on page 288. Explain how Sherlock Holmes used indirect proof, and include an example of indirect proof used every day.

STANDARDS PRACTICE

32. Theorem: Angles supplementary to the same angle are congruent.

Dia is proving the theorem above by contradiction. She began by assuming that $\angle A$ and $\angle B$ are supplementary to $\angle C$ and $\angle A \ncong \angle B$. Which of the following reasons will Dia use to reach a contradiction?

A If two angles form a linear pair, then they are supplementary angles.

B If two supplementary angles are equal, the angles each measure 90.

C The sum of the measures of the angles in a triangle is 180.

D If two angles are supplementary, the sum of their measures is 180.

33. REVIEW At a five-star restaurant, a waiter's total earnings t in dollars for working h hours in which he receives \$198 in tips is given by the following equation.

$$t = 2.5h + 198$$

If the waiter earned a total of \$213, how many hours did he work?

F 2 **H** 6

G 4 **J** 8

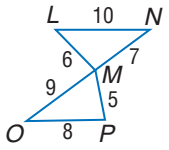
34. REVIEW Which expression has the *least* value?

A $|-28|$ **C** $|45|$

B $|15|$ **D** $|-39|$

Spiral Review

For Exercises 35 and 36, refer to the figure at the right. (Lesson 5-2)



35 Which angle in $\triangle MOP$ has the greatest measure?

36. Name the angle with the least measure in $\triangle LMN$.

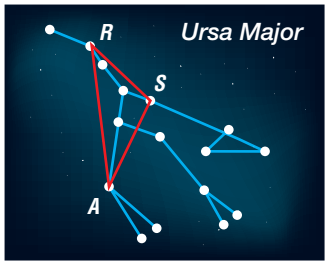
PROOF Write a two-column proof. (Lesson 5-1)

37. If an angle bisector of a triangle is also an altitude of the triangle, then the triangle is isosceles.

38. The median to the base of an isosceles triangle bisects the vertex angle.

39. Corresponding angle bisectors of congruent triangles are congruent.

40. ASTRONOMY Constellations were studied by astronomers to develop time-keeping systems. The Big Dipper is a part of the larger constellation Ursa Major. Three of the brighter stars in the constellation form $\triangle RSA$. If $m\angle R = 41$ and $m\angle S = 109$, find $m\angle A$. (Lesson 4-2)



GET READY for the Next Lesson

PREREQUISITE SKILL Determine whether each inequality is *true* or *false*.

41. $19 - 10 < 11$ **42.** $31 - 17 < 12$ **43.** $38 + 76 > 109$

CHAPTER 5

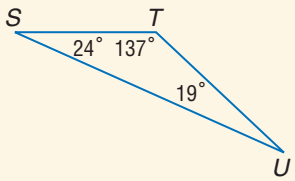
Mid-Chapter Quiz

Lessons 5-1 through 5-3

State whether each statement is *always*, *sometimes*, or *never* true. (Lesson 5-1)

1. The medians of a triangle intersect at one of the vertices of the triangle.
2. The angle bisectors of a triangle intersect at a point in the interior of the triangle.
3. The altitudes of a triangle intersect at a point in the exterior of the triangle.
4. The perpendicular bisectors of a triangle intersect at a point on the triangle.
5. Describe a triangle in which the angle bisectors all intersect in a point outside the triangle. If no triangle exists, write *no triangle*. (Lesson 5-1)

6. **MULTIPLE CHOICE** Which list gives the sides of $\triangle STU$ in order from longest to shortest? (Lesson 5-2)



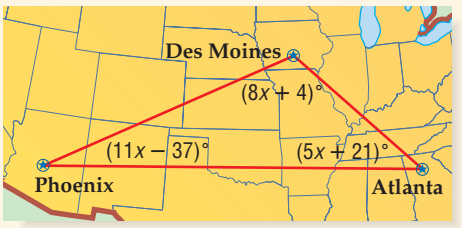
A $\overline{TU}, \overline{ST}, \overline{SU}$ C $\overline{SU}, \overline{ST}, \overline{TU}$

B $\overline{SU}, \overline{TU}, \overline{ST}$ D $\overline{ST}, \overline{TU}, \overline{SU}$

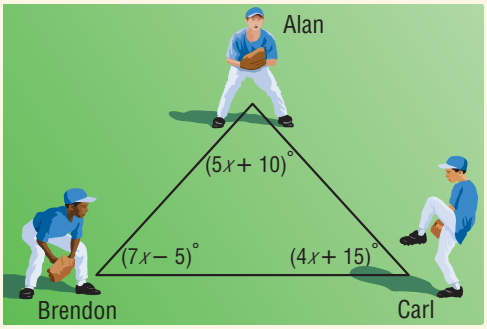
In $\triangle QRS$, $m\angle Q = x + 15$, $m\angle R = 2x + 10$, and $m\angle S = 4x + 15$. (Lesson 5-2)

7. Determine the measure of each angle.
8. List the sides in order from shortest to longest.

9. **TRAVEL** A plane travels from Des Moines to Phoenix, on to Atlanta, and then completes the trip directly back to Des Moines, as shown in the diagram. Write the lengths of the legs of the trip in order from greatest to least. (Lesson 5-2)



10. **BASEBALL** Alan, Brendon, and Carl were standing in the triangular shape shown below, throwing a baseball to warm up for a game. Between which two players was the throw the longest? (Lesson 5-2)

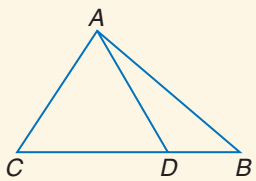


Write the assumption you would make to start an indirect proof of each statement. (Lesson 5-3)

11. The number 117 is divisible by 13.
12. $m\angle C < m\angle D$
13. n^3 is odd.
14. In a right triangle, $a^2 + b^2 = c^2$.
15. $\angle JKL \cong \angle WXY$
16. If n is an odd number, then $2n$ is an even number.
17. If $2x = 18$, then $x = 9$.

Write an indirect proof. (Lesson 5-3)

18. **Given:** $\triangle ABC$
    Prove: There can be no more than one obtuse angle in $\triangle ABC$.
19. **Given:** For lines m and n in plane $\mathcal{K}$, $m \nparallel n$.
    Prove: Lines m and n intersect at exactly one point.
20. **Given:** $m\angle ADC \neq m\angle ADB$
    Prove: $\overline{AD}$ is not an altitude of $\triangle ABC$.



EXPLORE 5-4

Graphing Calculator Lab

The Triangle Inequality

Standard 6.0 Students know and are able to use the triangle inequality theorem.

You can use the Cabri Junior application on a TI-83/84 Plus graphing calculator to discover properties of triangles.

ACTIVITY

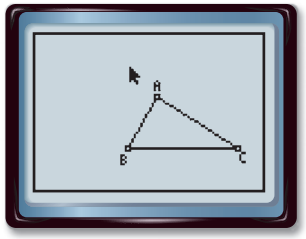
Construct a triangle. Observe the relationship between the sum of the lengths of two sides and the length of the other side.

Step 1 Construct a triangle using the triangle tool on the F2 menu. Then use the Alph-Num tool on the F5 menu to label the vertices as A, B, and C.

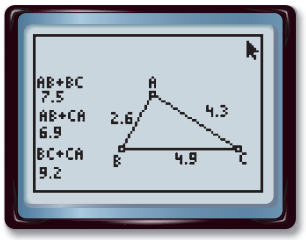
Step 2 Access the distance & length tool, shown as D. & Length, under Measure on the F5 menu. Use the tool to measure each side of the triangle.

Step 3 Display $AB + BC$, $AB + CA$, and $BC + CA$ by using the Calculate tool on the F5 menu. Label the measures.

Step 4 Click and drag the vertices to change the shape of the triangle.



Step 1



Steps 2 and 3

ANALYZE THE RESULTS

1. Replace each ● with $<$, $>$, or $=$ to make a true statement.
   $AB + BC$ ● CA $\quad$ $AB + CA$ ● BC $\quad$ $BC + CA$ ● AB
2. Click and drag the vertices to change the shape of the triangle. Then review your answers to Exercise 1. What do you observe?
3. Click on point A and drag it to lie on line BC. What do you observe about AB, BC, and CA? Are A, B, and C the vertices of a triangle? Explain.
4. **Make a conjecture** about the sum of the lengths of two sides of a triangle and the length of the third side.
5. Replace each ● with $<$, $>$, or $=$ to make a true statement.
   $|AB - BC|$ ● CA $\quad$ $|AB - CA|$ ● BC $\quad$ $|BC - CA|$ ● AB
   Then click and drag the vertices to change the shape of the triangle and review your answers. What do you observe?
6. How could you use your observations to determine the possible lengths of the third side of a triangle if you are given the lengths of the other two sides?

5-4 The Triangle Inequality

Main Ideas

- Apply the Triangle Inequality Theorem.
- Determine the shortest distance between a point and a line.

Standard 6.0 Students know and are able to use the triangle inequality theorem.

GET READY for the Lesson

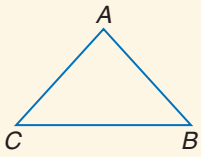
Chuck Noland travels between Los Angeles, Las Vegas, and Salt Lake City as part of his job. Mr. Noland lives in Los Angeles and needs to get to Salt Lake City as soon as possible. Should he take a flight that goes from Los Angeles to Salt Lake City, or a flight that goes from Los Angeles to Las Vegas, then to Salt Lake City?

The Triangle Inequality If you think Mr. Noland should fly directly from Los Angeles to Salt Lake City, you probably reasoned that a straight route is shorter. This is an example of the Triangle Inequality Theorem.

THEOREM 5.11 *Triangle Inequality Theorem*

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Examples:
$AB + BC > AC$
$BC + AC > AB$
$AC + AB > BC$



You will prove Theorem 5.11 in Exercise 21.

The Triangle Inequality Theorem can be used to determine whether three segments can form a triangle.

Study Tip

Inequality
If the sum of the least number and the middle number is greater than the greatest number, then each combination of inequalities is true.

EXAMPLE Identify Sides of a Triangle

1 **Determine whether the given measures can be the lengths of the sides of a triangle. 2, 4, 5**

Check each inequality.

$2 + 4 \stackrel{?}{>} 5$
$6 > 5$ ✓

$2 + 5 \stackrel{?}{>} 4$
$7 > 4$ ✓

$4 + 5 \stackrel{?}{>} 2$
$9 > 2$ ✓

All of the inequalities are true, so 2, 4, and 5 can be the lengths of the sides of a triangle.

CHECK Your Progress

1A. 6, 8, 14 **1B.** 8, 15, 17

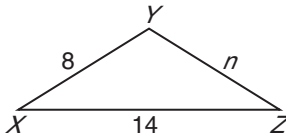
When you know the lengths of two sides of a triangle, you can determine the range of possible lengths for the third side.

STANDARDS EXAMPLE **Determine Possible Side Length**

2 Which of the following could *not* be the value of n?

A 6 **C** 14

B 10 **D** 18



Read the Item

You need to determine which value is not valid.

Test-Taking Tip

Testing Choices If you are short on time, you can test each choice to find the correct answer and eliminate any remaining choices.

Solve the Item

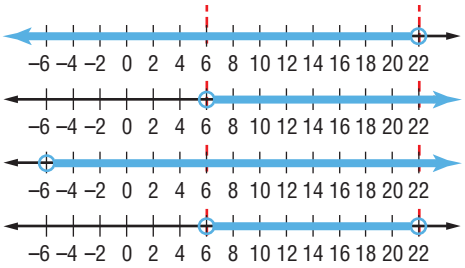
Solve each inequality to determine the range of values for YZ.

$XY + XZ > YZ$ $XY + YZ > XZ$ $YZ + XZ > XY$

$8 + 14 > n$ $8 + n > 14$ $n + 14 > 8$

$22 > n$ or $n < 22$ $n > 6$ $n > -6$

Graph the inequalities on the same number line.



The range of values that fit all three inequalities is $6 < n < 22$.

Examine the answer choices. The only value that does not satisfy the compound inequality is 6 since $6 = 6$. Thus, the answer is choice A.

CHECK Your Progress

2. If the measures of two sides of a triangle are 57 and 32, what is the *least* possible measure of the third side if the measure is an integer?

F 25 **G** 26 **H** 88 **J** 89

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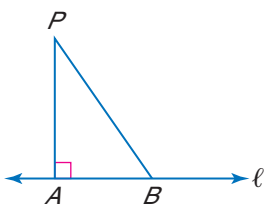
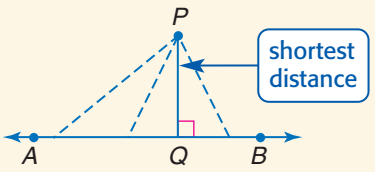
Distance Between a Point and a Line Recall that the distance between point P and line ℓ is measured along a perpendicular segment from the point to the line. It was accepted without proof that $\overline{PA}$ was the shortest segment from P to ℓ. The theorems involving the relationships between the angles and sides of a triangle can now be used to prove that a perpendicular segment is the shortest distance between a point and a line.



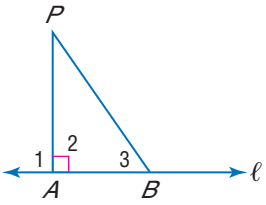
THEOREM 5.12

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Example: $\overline{PQ}$ is the shortest segment from P to $\overleftrightarrow{AB}$.



EXAMPLE 3 Prove Theorem 5.12



Given: $\overline{PA} \perp \ell$
$\overline{PB}$ is any nonperpendicular segment from P to ℓ.

Prove: $PB > PA$

Proof:

Statements	Reasons
1. $PA \perp \ell$	**1.** Given
2. $\angle 1$ and $\angle 2$ are right angles.	**2.** $\perp$ lines form right angles.
3. $\angle 1 \cong \angle 2$	**3.** All right angles are congruent.
4. $m\angle 1 = m\angle 2$	**4.** Definition of congruent angles
5. $m\angle 1 > m\angle 3$	**5.** Exterior Angle Inequality Theorem
6. $m\angle 2 > m\angle 3$	**6.** Substitution Property
7. $PB > PA$	**7.** If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

Study Tip

Shortest Distance to a Line

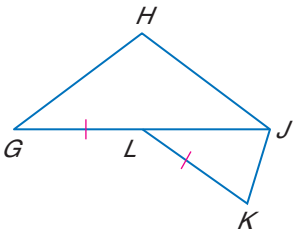
If a line is horizontal, the shortest distance from a point to that line will be along a vertical line. Likewise, the shortest distance from a point to a vertical line lies along a horizontal line.

CHECK Your Progress

3. Write a two-column proof.

Given: $GL = LK$

Prove: $JH + GH > JK$

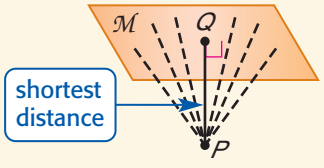


Corollary 5.1 follows directly from Theorem 5.12.

COROLLARY 5.1

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Example: $\overline{QP}$ is the shortest segment from P to Plane $\mathcal{M}$.



You will prove Corollary 5.1 in Exercise 6.

CHECK Your Understanding

Example 1 (p. 296)

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

1. 5, 4, 3
2. 5, 15, 10
3. 30.1, 0.8, 31
4. 5.6, 10.1, 5.2

Example 2 (p. 297)

5. **STANDARDS PRACTICE** An isosceles triangle has a base 10 units long. If the congruent sides have whole number measures, what is the *least* possible length of the sides?

A 5 **B** 6 **C** 17 **D** 21

Example 3 (p. 298)

6. **PROOF** Write a proof for Corollary 5.1.
Given: $\overline{PQ} \perp$ plane $\mathcal{M}$
Prove: $\overline{PQ}$ is the shortest segment from P to plane $\mathcal{M}$.

Exercises

HOMEWORK HELP

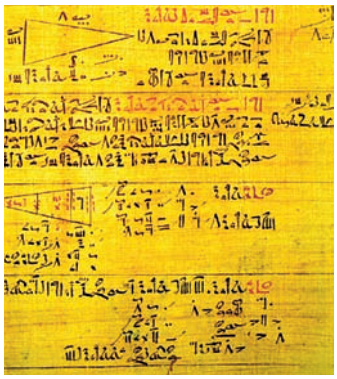
For Exercises	See Examples
7–12	1
13–18	2
19–20	3

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

7. 1, 2, 3
8. 2, 6, 11
9. 8, 8, 15
10. 13, 16, 29
11. 18, 32, 21
12. 9, 21, 20

Find the range for the measure of the third side of a triangle given the measures of two sides.

13. 5 and 11
14. 7 and 9
15. 10 and 15
16. 12 and 18
17. 21 and 47
18. 32 and 61

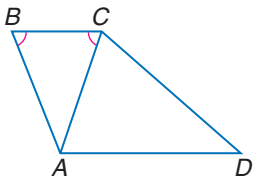


Real-World Link
Ancient Egyptians used pieces of flattened, dried papyrus reed as paper. From surviving examples like the Rhind Papyrus and the Moscow Papyrus, we have learned a bit about Egyptian mathematics.

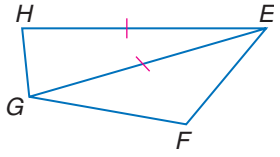
Source: aldokkan.com

PROOF **Write a two-column proof.**

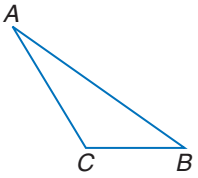
19. **Given:** $\angle B \cong \angle ACB$
Prove: $AD + AB > CD$



20. **Given:** $\overline{HE} \cong \overline{EG}$
Prove: $HE + FG > EF$



21. **Given:** $\triangle ABC$
Prove: $AC + BC > AB$ (Triangle Inequality Theorem)
(*Hint:* Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



22. **HISTORY** The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. How many different triangles can be formed using the rope below?

ALGEBRA **Determine whether the given coordinates are the vertices of a triangle. Explain.**

23. $A(5, 8)$, $B(2, -4)$, $C(-3, -1)$

24. $L(-24, -19)$, $M(-22, 20)$, $N(-5, -7)$

25. $X(0, -8)$, $Y(16, -12)$, $Z(28, -15)$

26. $R(1, -4)$, $S(-3, -20)$, $T(5, 12)$

SCRAPBOOKING **For Exercises 27 and 28, use the following information.** Carlota has several strips of trim she wishes to use as a triangular border for a spread in her scrapbook. The strips measure 3 centimeters, 4 centimeters, 5 centimeters, 6 centimeters, and 12 centimeters.

27. How many different triangles could Carlota make with the strips?

28. How many different triangles could Carlota make that have a perimeter that is divisible by 3?

PROBABILITY **For Exercises 29 and 30, use the following information.** One side of a triangle is 2 feet long. Let m represent the measure of the second side and n represent the measure of the third side. Suppose m and n are whole numbers and that $14 < m < 17$ and $13 < n < 17$.

29. List the measures of the sides of the triangles that are possible.

30. What is the probability that a randomly chosen triangle that satisfies the given conditions will be isosceles?

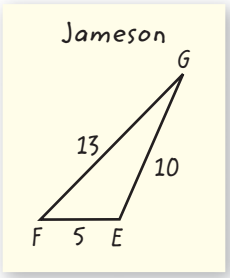
EXTRA PRACTICE
See pages 810, 832.
Math Online
Self-Check Quiz at
ca.geometryonline.com

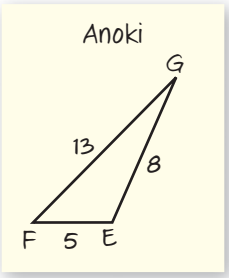
H.O.T. Problems

31. **REASONING** Explain why the distance between two nonhorizontal parallel lines on a coordinate plane cannot be found using the distance between their y-intercepts.

32. **OPEN ENDED** Find three numbers that can be the lengths of the sides of a triangle and three numbers that cannot be the lengths of the sides of a triangle. Justify your reasoning with a drawing.

33. **FIND THE ERROR** Jameson and Anoki drew $\triangle EFG$ with $FG = 13$ and $EF = 5$. Each chose a possible measure for $\overline{GE}$. Who is correct? Explain.



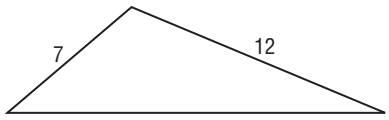


34. **CHALLENGE** State and prove a theorem that compares the measures of each side of a triangle with the differences of the measures of the other two sides.

35. ***Writing in Math*** Refer to the information on page 296. Explain why it is not always possible to apply the Triangle Inequality Theorem when traveling.

STANDARDS PRACTICE

36. If two sides of a triangle measure 12 and 7, which of the following can *not* be the perimeter of the triangle?



A 29 **C** 37

B 34 **D** 38

37. REVIEW Which equation describes the line that passes through the point (5, 3) and is parallel to the line represented by the equation $-2x + y = -4$?

F $y = \frac{1}{2}x + 5.5$

G $y = 2x - 7$

H $y = -2x + 13$

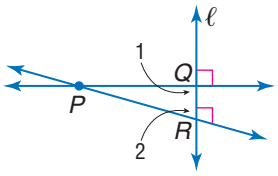
J $y = \frac{2}{3}x + 15$

Spiral Review

38. PROOF Write an indirect proof. (Lesson 5-3)

Given: P is a point not on line ℓ.

Prove: $\overline{PQ}$ is the only line through P perpendicular to ℓ.



39. TRAVEL Maddie drove 175 miles from Seattle, Washington, to Portland, Oregon. It took her three hours to complete the trip. Prove that her average driving speed was less than 60 miles per hour. (Lesson 5-3)

ALGEBRA List the sides of $\triangle PQR$ in order from longest to shortest if the angles of $\triangle PQR$ have the given measures. (Lesson 5-2)

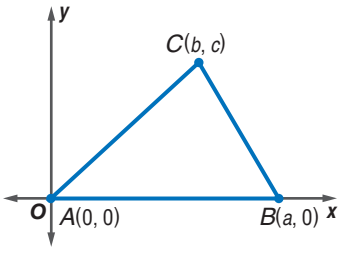
40. $m\angle P = 7x + 8, m\angle Q = 8x - 10, m\angle R = 7x + 6$

41. $m\angle P = 3x + 44, m\angle Q = 68 - 3x, m\angle R = x + 61$

For Exercises 42 and 43, refer to the figure. (Lesson 4-7)

42. Find the coordinates of D if the x-coordinate of D is the mean of the x-coordinates of the vertices of $\triangle ABC$ and the y-coordinate is the mean of the y-coordinates of the vertices of $\triangle ABC$.

43. Prove that D is the intersection of the medians of $\triangle ABC$.



Determine whether $\triangle JKL \cong \triangle PQR$ given the coordinates of the vertices. Explain. (Lesson 4-4)

44. $J(0, 5), K(0, 0), L(-2, 0), P(4, 8), Q(4, 3), R(6, 3)$

45. $J(6, 4), K(1, -6), L(-9, 5), P(0, 7), Q(5, -3), R(15, 8)$

46. $J(-6, -3), K(1, 5), L(2, -2), P(2, -11), Q(5, -4), R(10, -10)$

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each inequality. (Pages 783–784)

47. $3x + 54 < 90$ **48.** $8x - 14 < 3x + 19$ **49.** $4x + 7 < 180$

5-5 Inequalities Involving Two Triangles

Main Ideas

- Apply the SAS Inequality.
- Apply the SSS Inequality.

Standard 6.0 **Students** know and **are able to use the triangle inequality theorem.**

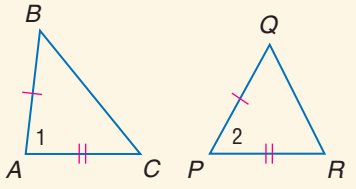
GET READY for the Lesson

Many objects have a fixed arm connected with a joint or hinge to a second arm or stand. This thrill ride at Cedar Point in Sandusky, Ohio, sends riders into the sky in a pendulum motion. As the pendulum rises, the angle between the arm and the legs of the stand decreases until the arm moves past the stand. Then the angle increases. The distance between the riders and the docking station changes as the angle changes.

SAS Inequality The relationship of the arms and the angle between them illustrates the following theorem.

THEOREM 5.13 **SAS Inequality/Hinge Theorem**

Two sides of a triangle are congruent to two sides of another triangle. If the included angle in the first triangle has a greater measure than the included angle in the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

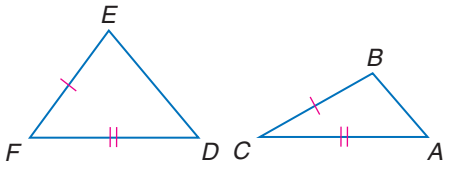


Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $m\angle 1 > m\angle 2$, then $BC > QR$.

PROOF **SAS Inequality Theorem**

Given: $\triangle ABC$ and $\triangle DEF$
$\overline{AC} \cong \overline{DF}$, $\overline{BC} \cong \overline{EF}$
$m\angle F > m\angle C$

Prove: $DE > AB$



Proof:

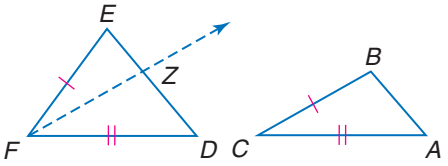
We are given that $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$. We also know that $m\angle F > m\angle C$. Draw auxiliary ray FZ such that $m\angle DFZ = m\angle C$ and that $\overline{ZF} \cong \overline{BC}$. This leads to two cases.

Study Tip

SAS Inequality

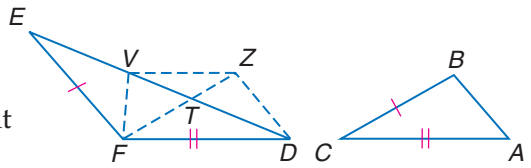
The SAS Inequality Theorem is also called the *Hinge Theorem.*

Case 1: If Z lies on $\overline{DE}$, then $\triangle FZD \cong \triangle CBA$ by SAS. Thus, $ZD = BA$ by CPCTC and the definition of congruent segments.



By the Segment Addition Postulate, $DE = EZ + ZD$. Also, $DE > ZD$ by the definition of inequality. Therefore, $DE > AB$ by the Substitution Property.

Case 2: If Z does not lie on $\overline{DE}$, then let the intersection of $\overline{FZ}$ and $\overline{ED}$ be point T. Now draw another auxiliary segment $\overline{FV}$ such that V is on $\overline{DE}$ and $\angle EFV \cong \angle VFZ$.



Since $\overline{FZ} \cong \overline{BC}$ and $\overline{BC} \cong \overline{EF}$, we have $\overline{FZ} \cong \overline{EF}$ by the Transitive Property. Also $\overline{VF}$ is congruent to itself by the Reflexive Property. Thus, $\triangle EFV \cong \triangle ZFV$ by SAS. By CPCTC, $\overline{EV} \cong \overline{ZV}$ or $EV = ZV$. Also, $\triangle FZD \cong \triangle CBA$ by SAS. So, $\overline{ZD} \cong \overline{BA}$ by CPCTC or $ZD = BA$.

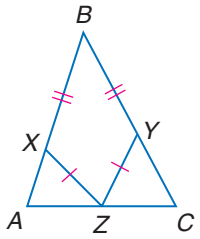
In $\triangle VZD$, $VD + ZV > ZD$ by the Triangle Inequality Theorem. By substitution, $VD + EV > ZD$. Since $ED = VD + EV$ by the Segment Addition Postulate, $ED > ZD$. Using substitution, $ED > BA$ or $DE > AB$.

EXAMPLE Use SAS Inequality in a Proof

1 **Write a two-column proof.**

Given: $\overline{YZ} \cong \overline{XZ}$
Z is the midpoint of $\overline{AC}$.
$m\angle CZY > m\angle AZX$
$\overline{BY} \cong \overline{BX}$

Prove: $BC > AB$



Proof:

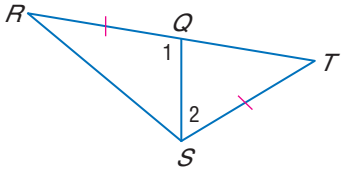
Statements	Reasons
1. $\overline{YZ} \cong \overline{XZ}$, Z is the midpoint of $\overline{AC}$, $m\angle CZY > m\angle AZX$, $\overline{BY} \cong \overline{BX}$	**1.** Given
2. $CZ = AZ$	**2.** Definition of midpoint
3. $CY > AX$	**3.** SAS Inequality
4. $BY = BX$	**4.** Definition of congruent segments
5. $CY + BY > AX + BX$	**5.** Addition Property
6. $BC = CY + BY$, $AB = AX + BX$	**6.** Segment Addition Postulate
7. $BC > AB$	**7.** Substitution Property

CHECK Your Progress

1. Write a two-column proof.

Given: $\overline{RQ} \cong \overline{ST}$

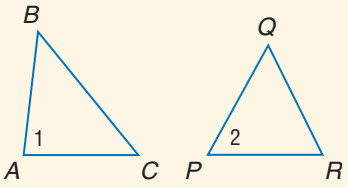
Prove: $RS > TQ$



SSS Inequality The converse of the SAS Inequality Theorem is the SSS Inequality Theorem.

THEOREM 5.14 **SSS Inequality Theorem**

If two sides of a triangle are congruent to two sides of another triangle and the third side in one triangle is longer than the third side in the other, then the angle between the pair of congruent sides in the first triangle is greater than the corresponding angle in the second triangle.



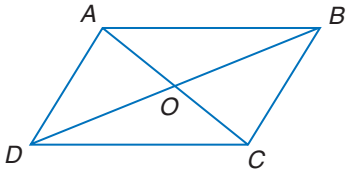
Example: Given $\overline{AB} \cong \overline{PQ}$, $\overline{AC} \cong \overline{PR}$, if $BC > QR$, then $m\angle 1 > m\angle 2$.

You will prove Theorem 5.14 in Exercise 24.

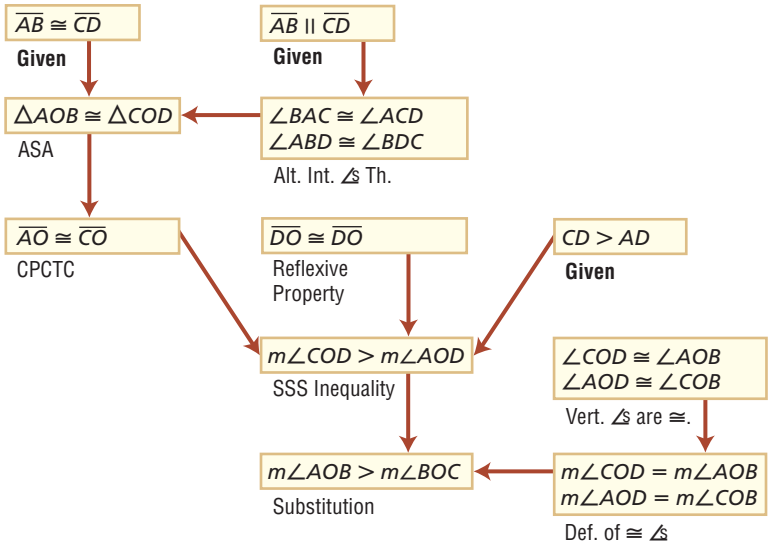
EXAMPLE 2 Prove Triangle Relationships

Given: $\overline{AB} \cong \overline{CD}$
$\overline{AB} \parallel \overline{CD}$
$CD > AD$

Prove: $m\angle AOB > m\angle BOC$



Flow Proof:



Study Tip

Proofs

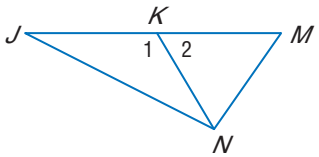
Check each step in your proof. Make sure that each statement has a reason. Each statement should follow logically from the previous or given statements.

CHECK Your Progress

2. Write a two-column proof.

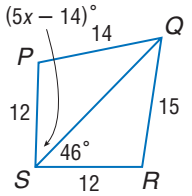
Given: $\overline{NK}$ is a median of $\triangle JMN$.
$JN > NM$

Prove: $m\angle 1 > m\angle 2$



EXAMPLE 3 Relationships Between Two Triangles

ALGEBRA **Write an inequality using the information in the figure.**



a. Compare $m\angle QSR$ and $m\angle QSP$.

In $\triangle PQS$ and $\triangle RQS$, $\overline{PS} \cong \overline{RS}$, $\overline{QS} \cong \overline{QS}$, and $QR > QP$. The SAS Inequality allows us to conclude that $m\angle QSR > m\angle QSP$.

b. Find the range of values containing x.

By the SSS Inequality, $m\angle QSR > m\angle QSP$, or $m\angle QSP < m\angle QSR$.

$m\angle QSP < m\angle QSR$ SSS Inequality

$5x - 14 < 46$ Substitution

$5x < 60$ Add 14 to each side.

$x < 12$ Divide each side by 5.

Also, recall that the measure of any angle is always greater than 0.

$5x - 14 > 0$

$5x > 14$ Add 14 to each side.

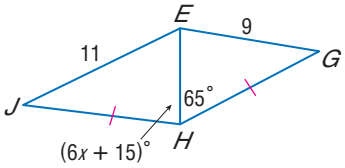
$x > \frac{14}{5}$ or 2.8 Divide each side by 5.

The two inequalities can be written as the compound inequality $2.8 < x < 12$.

CHECK Your Progress

3A. Write an inequality to compare $m\angle JHE$ and $m\angle GHE$.

3B. Find the range of values containing x.



Online **Personal Tutor at** ca.geometryonline.com

Real-World Link

Physical therapists help their patients regain range of motion after an illness or injury.

Source: www.apta.org

EXAMPLE 4 Use Triangle Inequalities

HEALTH ***Range of motion*** **describes how much a limb can be moved from a straight position. To determine the range of motion of a person's arm, determine the distance from the wrist to the shoulder when the elbow is bent as far as possible.**

Jessica can bend her left arm so her left wrist is 5 inches from her shoulder and her right arm so her right wrist is 3 inches from her shoulder. Which arm has the greater range of motion? Explain.

The distance between the wrist and shoulder is smaller on her right arm. Assuming that both arms have the same measurements, the SSS inequality tells us that the angle formed at the elbow is smaller on the right arm. This means that the right arm has a greater range of motion.

CHECK Your Progress

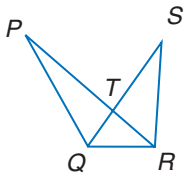
4. After physical therapy, Jessica can bend her left arm so her left wrist is 2 inches from her shoulder. She can bend her right arm so her right wrist is $2\frac{1}{2}$ inches from her shoulder. Which arm has the better range of motion now? Explain.

CHECK Your Understanding

PROOF Write a two-column proof.

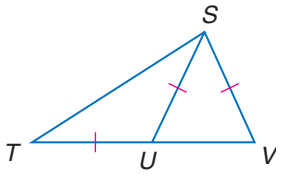
Example 1 (p. 303)

1. Given: $\overline{PQ} \cong \overline{SQ}$
Prove: $PR > SR$



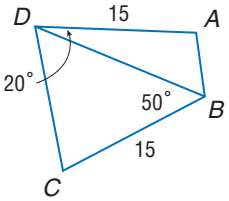
Example 2 (p. 304)

2. Given: $\overline{TU} \cong \overline{US}$
$\overline{US} \cong \overline{SV}$
Prove: $ST > UV$

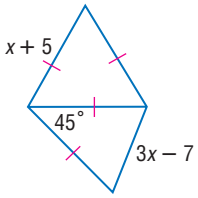


Example 3 (p. 305)

3. Write an inequality relating AB and CD.

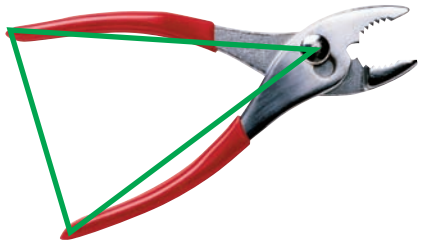


4. Write an inequality to describe the possible values of x.



Example 4 (p. 305)

5. PHYSICAL SCIENCE A lever is used to multiply the force applied to an object. One example of a lever is a pair of pliers. Use the SAS or SSS Inequality to explain how to use a pair of pliers.

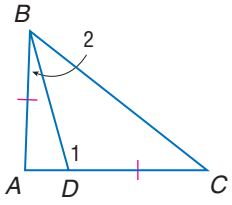


Exercises

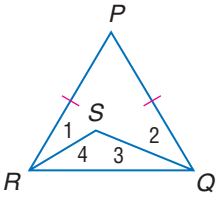
HOMEWORK HELP	
For Exercises	See Examples
6, 7	1
8, 9	2
10–15	3
16, 17	4

PROOF Write a two-column proof.

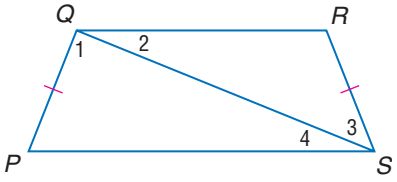
6. Given: $\triangle ABC$
$\overline{AB} \cong \overline{CD}$
Prove: $BC > AD$



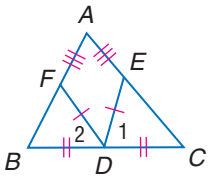
7. Given: $\overline{PR} \cong \overline{PQ}$
$SQ > SR$
Prove: $m\angle 1 < m\angle 2$

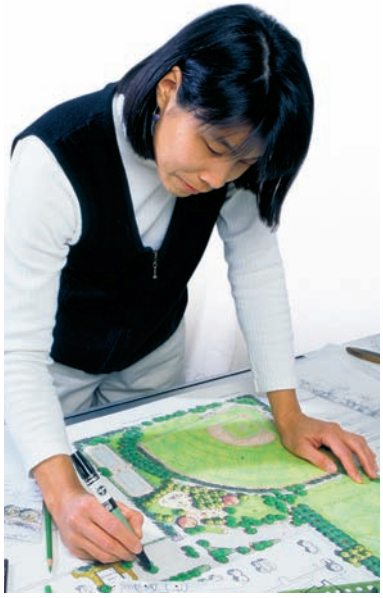


8. Given: $\overline{PQ} \cong \overline{RS}$
$QR < PS$
Prove: $m\angle 3 < m\angle 1$



9. Given: $\overline{ED} \cong \overline{DF}$
$m\angle 1 > m\angle 2$
D is the midpoint of $\overline{CB}$.
$\overline{AE} \cong \overline{AF}$
Prove: $AC > AB$



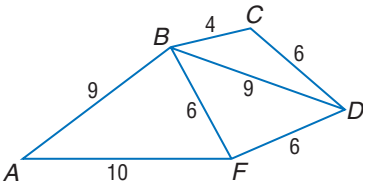


Write an inequality relating the given pair of angles or segment measures.

10. AB, FD

11. $m\angle BDC$, $m\angle FDB$

12. $m\angle FBA$, $m\angle DBF$

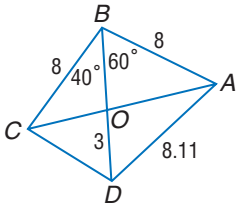


Write an inequality relating the given pair of angles or segment measures.

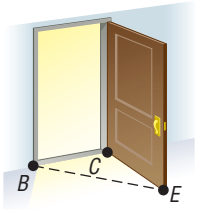
13. AD, DC

14. OC, OA

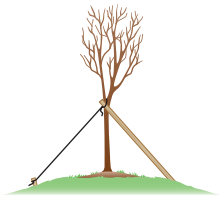
15. $m\angle AOD$, $m\angle AOB$



16. DOORS Open a door slightly. With the door open, measure the angle made by the door and the door frame. Measure the distance from the end of the door to the door frame. Open the door wider, and measure again. How do the measures compare?

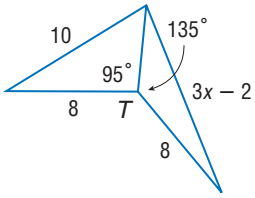


17. LANDSCAPING When landscapers plant new trees, they usually brace the tree using a stake tied to the trunk of the tree. Use the SAS or SSS Inequality to explain why this is an effective method for keeping a newly planted tree perpendicular to the ground .

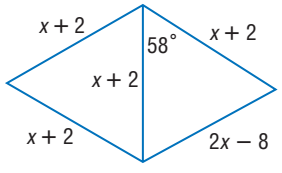


Write an inequality to describe the possible values of x.

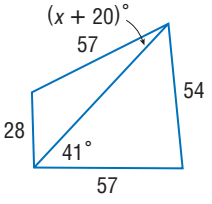
18.



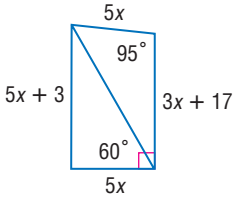
19.



20.



21.

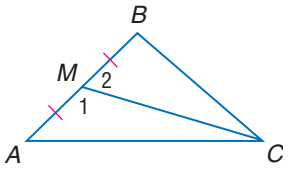


Real-World Career
Landscape Architect
Landscape architects design the settings of buildings and parks. By arranging the locations of the buildings and the plants, they make the site functional, beautiful, and environmentally friendly.

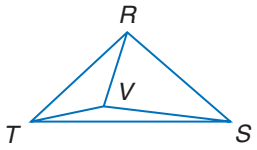
MathOnline
For more information, go to ca.geometryonline.com.

Write an inequality to describe the possible values of x.

22. In the figure, $\overline{AM} \cong \overline{MB}$, $AC > BC$, $m\angle 1 = 5x + 20$ and $m\angle 2 = 8x - 100$.



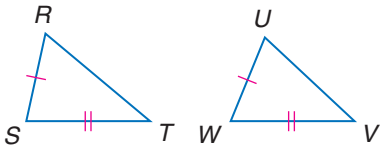
23. In the figure, $m\angle RVS = 15 + 5x$, $m\angle SVT = 10x - 20$, $RS < ST$, and $\angle RTV \cong \angle TRV$.



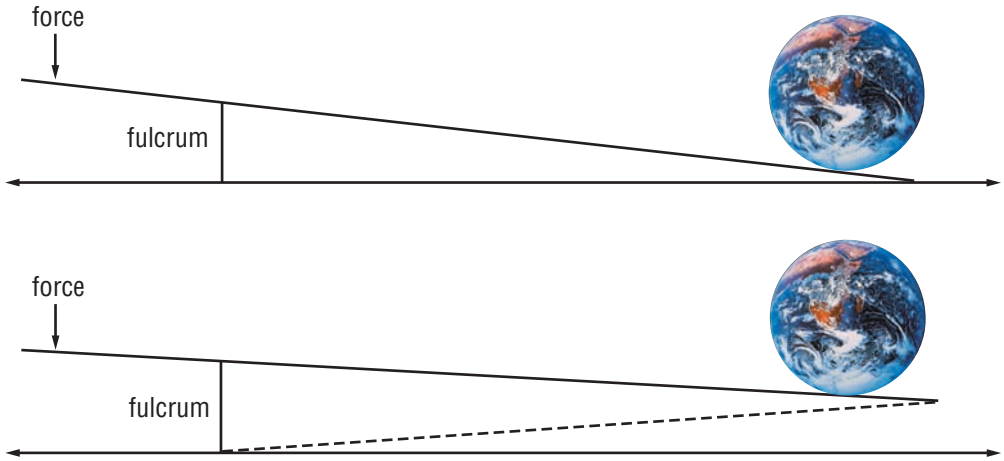
24. PROOF Use an indirect proof to prove the SSS Inequality Theorem (Theorem 5.14).

Given: $\overline{RS} \cong \overline{UW}$
$\overline{ST} \cong \overline{WV}$
$RT > UV$

Prove: $m\angle S > m\angle W$



25. HISTORY When force is applied to a lever that is balanced on a fulcrum, you can lift a heavy object. In the third century, Archimedes said, "Give me a place to stand and a lever long enough, and I will move the Earth." Write a description of how the triangle formed from the height of the fulcrum and the length of the lever from the fulcrum to Earth applies the SAS Inequality Theorem.



EXTRA PRACTICE
See pages 810, 832.
Math Online
Self-Check Quiz at
ca.geometryonline.com

H.O.T. Problems

26. OPEN ENDED Describe a real-world object that illustrates either the SAS or the SSS inequality.

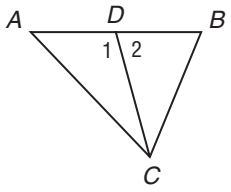
27. REASONING Compare and contrast the SSS Inequality Theorem to the SSS Postulate for triangle congruence.

28. CHALLENGE The SAS Inequality states that the base of an isosceles triangle gets longer as the measure of the vertex angle increases. Describe the effect of changing the measure of the vertex angle on the measure of the altitude. Justify your answer.

29. *Writing in Math* Refer to the information on page 302. Write a description of the angle between the arm and the stand as the ride operator lifts and then lowers the pendulum. Include an explanation of how the distance between the ends of the arm and stand is related to the angle between them.

STANDARDS PRACTICE

30. If $\overline{DC}$ is a median of $\triangle ABC$ and $m\angle 1 > m\angle 2$, which of the following statements is *not* true?



A $AD = BD$

B $m\angle ADC = m\angle BDC$

C $AC > BC$

D $m\angle 1 > m\angle B$

31. REVIEW The weight of an object on Jupiter varies directly with its weight on Earth. If an object that weighs 5 pounds on Earth weighs 11.5 pounds on Jupiter, how much will a 7-pound object weigh on Jupiter?

F 9.3 pounds

G 13.5 pounds

H 16.1 pounds

J 80.5 pounds

Spiral Review

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain. (Lesson 5-4)

32. 25, 1, 21 **33.** 16, 6, 19 **34.** 8, 7, 15

Write the assumption you would make to start an indirect proof of each statement. (Lesson 5-3)

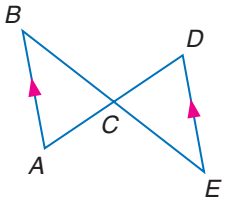
35. $\overline{AD}$ is a median of $\triangle ABC$.

36. If two altitudes of a triangle are congruent, then the triangle is isosceles.

PROOF Write a two-column proof. (Lesson 4-5)

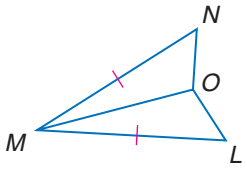
37. Given: $\overline{AD}$ bisects $\overline{BE}$. $\overline{AB} \parallel \overline{DE}$

Prove: $\triangle ABC \cong \triangle DEC$



38. Given: $\overline{OM}$ bisects $\angle LMN$. $\overline{LM} \cong \overline{MN}$

Prove: $\triangle MOL \cong \triangle MON$



Find the measures of the sides of $\triangle EFG$ with the given vertices and classify each triangle by its sides. (Lesson 4-1)

39. $E(4, 6)$, $F(4, 11)$, $G(9, 6)$

40. $E(-7, 10)$, $F(15, 0)$, $G(-2, -1)$

41. $E(16, 14)$, $F(7, 6)$, $G(-5, -14)$

42. $E(9, 9)$, $F(12, 14)$, $G(14, 6)$

Write an equation in point-slope form of the line having the given slope that contains the given point. (Lesson 3-4)

43. $m = 2$, $(4, 3)$ **44.** $m = -3$, $(2, -2)$ **45.** $m = 11$, $(-4, -9)$

46. ADVERTISING An ad for Wildflowers Gift Boutique says *When it has to be special, it has to be Wildflowers.* Catalina needs a special gift. Does it follow that she should go to Wildflowers? Explain. (Lesson 2-4)

CHAPTER 5

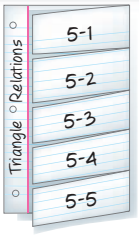
Study Guide and Review

STUDY TO GO Download Vocabulary Review from ca.geometryonline.com

FOLDABLES™ Study Organizer

GET READY to Study

Be sure the following Key Concepts are noted in your Foldable.



Key Concepts

Special Segments in Triangles (Lesson 5-1)

- The special segments of triangles are perpendicular bisectors, angle bisectors, medians, and altitudes.
- The intersection points of each of the special segments of a triangle are called the *points of concurrency.*
- The points of concurrency for a triangle are the circumcenter, incenter, centroid, and orthocenter.

Indirect Proof (Lesson 5-3)

- Writing an Indirect Proof:
  1. Assume that the conclusion is false.
  2. Show that this assumption leads to a contradiction.
  3. Since the false conclusion leads to an incorrect statement, the original conclusion must be true.

Triangle Inequalities (Lessons 5-2, 5-4, 5-5)

- The largest angle in a triangle is opposite the longest side, and the smallest angle is opposite the shortest side.
- The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
- SAS Inequality (Hinge Theorem): In two triangles, if two sides are congruent, then the measure of the included angle determines which triangle has the longer third side.
- SSS Inequality: In two triangles, if two corresponding sides of each triangle are congruent, then the length of the third side determines which triangle has the included angle with the greater measure.

Key Vocabulary

altitude (p. 272)
centroid (p. 271)
circumcenter (p. 270)
concurrent lines (p. 270)
incenter (p. 271)
indirect proof (p. 288)
indirect reasoning (p. 288)
median (p. 271)
orthocenter (p. 272)
perpendicular bisector (p. 269)
point of concurrency (p. 270)
proof by contradiction (p. 288)

Vocabulary Check

Choose the correct term to complete each sentence.

1. All of the angle bisectors of a triangle meet at the (incenter, circumcenter).
2. In $\triangle RST$, if point P is the midpoint of $\overline{RS}$, then $\overline{PT}$ is a(n) (angle bisector, median).
3. The theorem that the sum of the lengths of two sides of a triangle is greater than the length of the third side is the (Triangle Inequality Theorem, SSS Inequality).
4. The three medians of a triangle intersect at the (centroid, orthocenter).
5. In $\triangle JKL$, if point H is equidistant from $\overrightarrow{KJ}$ and $\overrightarrow{KL}$, then $\overleftrightarrow{HK}$ is an (angle bisector, altitude).
6. The circumcenter of a triangle is the point where all three (perpendicular bisectors, medians) of the sides of the triangle intersect.
7. In $\triangle ABC$, if $\overleftrightarrow{AK} \perp \overleftrightarrow{BC}$, $\overleftrightarrow{BK} \perp \overleftrightarrow{AC}$, and $\overleftrightarrow{CK} \perp \overleftrightarrow{AB}$, then K is the (orthocenter, incenter) of $\triangle ABC$.
8. In writing an indirect proof, begin by assuming that the (hypothesis, conclusion) is false.

Math Online Vocabulary Review at ca.geometryonline.com

Mixed Problem Solving
For mixed problem-solving practice, see page 832.

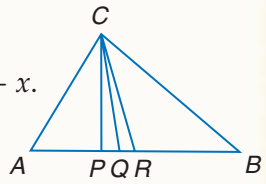
Lesson-by-Lesson Review

5-1 Bisectors, Medians, and Altitudes (pp. 269–278)

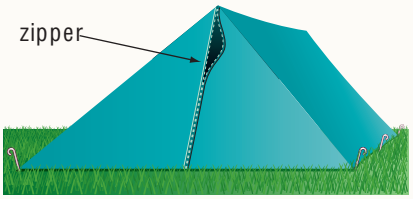
In the figure, $\overline{CP}$ is an altitude, $\overline{CQ}$ is the angle bisector of $\angle ACB$, and R is the midpoint of $\overline{AB}$.

9. Find $m\angle ACQ$ if $m\angle ACB = 123 - x$ and $m\angle QCB = 42 + x$.

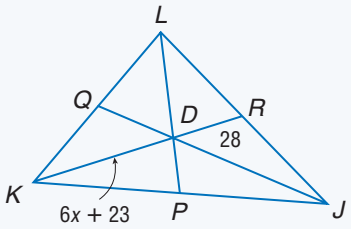
10. Find AB if $AR = 3x + 6$ and $RB = 5x - 14$.



11. TENT DESIGN Kame created a design for a new tent. She placed a zipper that extended from the midpoint of the base of one triangular face of the tent all the way to the top of the tent, as shown. Which special segment of triangles could represent this zipper?



Example 1 **Points P, Q, and R are the midpoints of $\overline{JK}$, $\overline{KL}$, and $\overline{JL}$, respectively. Find x.**



$KD = \frac{2}{3}(KR)$	Centroid Theorem
$6x + 23 = \frac{2}{3}(6x + 51)$	Substitution
$6x + 23 = 4x + 34$	Simplify.
$2x = 11$	Subtract $4x + 23$ from each side.
$x = \frac{11}{2}$	Divide each side by 2.

5-2 Inequalities and Triangles (pp. 280–287)

Use the figure in Example 2 to determine the relationship between the lengths of the given sides.

12. $\overline{SR}$, $\overline{SD}$

13. $\overline{DQ}$, $\overline{DR}$

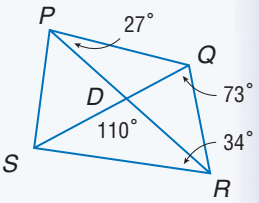
14. $\overline{PQ}$, $\overline{QR}$

15. $\overline{SR}$, $\overline{SQ}$

16. COORDINATE GEOMETRY Triangle WXY has vertices $W(2, 1)$, $X(-1, -2)$, and $Y(3, -4)$. List the angles in order from the least to the greatest measure.

Example 2 **Determine the relationship between the lengths of $\overline{SD}$ and $\overline{QD}$.**

$\overline{SD}$ is opposite $\angle SRD$. $\overline{QD}$ is opposite $\angle QRD$. Since $m\angle QDR = 70$ by the Supplement Theorem, and $m\angle QRD = 37$ by the Angle Sum Theorem, then $m\angle SRD < m\angle QRD$. Therefore, $SD < QD$.



CHAPTER 5

Study Guide and Review

5-3 Indirect Proof (pp. 288–293)

17. FOOTBALL Gabriel plays quarterback for his high school team. This year, he completed 101 passes in the five games in which he played. Prove that, in at least one game, Gabriel completed more than 20 passes.

Example 3 **State the assumption you would make to start an indirect proof of the statement *If $3x + 1 > 10$, then $x > 3$.***

The conclusion of the conditional statement is $x > 3$. The negation of the conclusion is $x \leq 3$.

5-4 The Triangle Inequality (pp. 296–301)

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

18. 7, 20, 5

19. 16, 20, 5

20. 18, 20, 6

21. 19, 19, 41

22. GARDENING James has three garden timbers that measure 2 feet, 3 feet, and 6 feet long. Could these be used to enclose a triangular garden? Explain.

Example 4 **Determine whether 7, 6, and 14 can be the lengths of the sides of a triangle.**

Check each inequality.

$7 + 6 \stackrel{?}{>} 14$
$13 \not> 14$ False

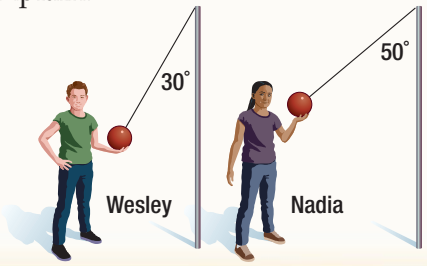
$7 + 14 \stackrel{?}{>} 6$
$21 > 6$ True

$6 + 14 \stackrel{?}{>} 7$
$20 > 7$ True

Because the inequalities are not true in all cases, the sides cannot form a triangle.

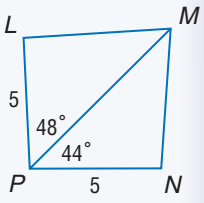
5-5 Inequalities Involving Two Triangles (pp. 302–309)

23. SPORTS Wesley and Nadia are playing tetherball. The photo shows them at two different points in the game. Who was standing closer to the pole? Explain.



Example 5 **Write an inequality relating *LM* and *MN*.**

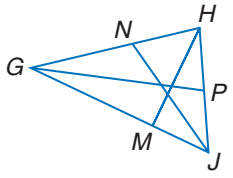
In $\triangle LMP$ and $\triangle NMP$, $\overline{LP} \cong \overline{NP}$, $\overline{PM} \cong \overline{PM}$, and $m\angle LPM > m\angle NPM$. The SAS Inequality allows us to conclude that $LM > MN$.



CHAPTER 5

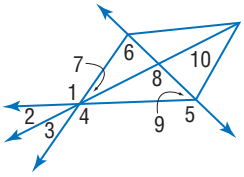
Practice Test

In $\triangle GHJ$, $HP = 5x - 16$, $PJ = 3x + 8$, $m\angle GJN = 6y - 3$, $m\angle NJH = 4y + 23$, and $m\angle HMG = 4z + 14$.



1. $\overline{GP}$ is a median of $\triangle GHJ$. Find HJ.
2. Find $m\angle GJH$ if $\overline{JN}$ is an angle bisector.
3. If $\overline{HM}$ is an altitude of $\triangle GHJ$, find the value of z.

Refer to the figure below. Determine which angle in each set has the greatest measure.



4. $\angle 8$, $\angle 5$, $\angle 7$
5. $\angle 6$, $\angle 7$, $\angle 8$
6. $\angle 1$, $\angle 6$, $\angle 9$

Write the assumption you would make to start an indirect proof of each statement.

7. If n is a natural number, then $2n + 1$ is odd.
8. Alternate interior angles are congruent.

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

9. 7, 24, 25
10. 25, 35, 60
11. 20, 3, 18
12. 5, 10, 6

13. **DESIGN** A landscape architect is making a model of a site. If the lengths of rods are 4 inches, 6 inches, and 8 inches, can these rods form a triangle? Explain.

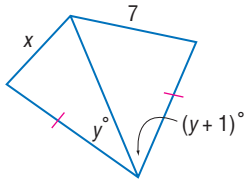
14. **BUSINESS** Over the course of three days, Marcus spent one and a half hours in a teleconference for his marketing firm. Use indirect reasoning to show that, on at least one day, Marcus spent at least a half hour in a teleconference.

Find the range for the measure of the third side of a triangle given the measures of two sides.

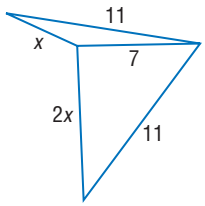
15. 1 and 14
16. 14 and 11

Write an inequality for the possible values of x.

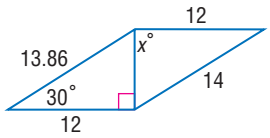
17.



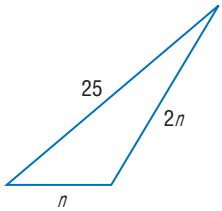
18.



19.



20. **MULTIPLE CHOICE** In the figure below, n is a whole number. What is the *least* possible value for n?



A 8
B 9
C 11
D 24

CHAPTER 5

California Standards Practice

Cumulative, Chapters 1–5

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1 Which of the following is a logical conclusion based on the statement and its converse below?

Statement: If the measure of an angle is 50°, then the angle is an acute angle.

Converse: If an angle is an acute angle, then the measure of the angle is 50°.

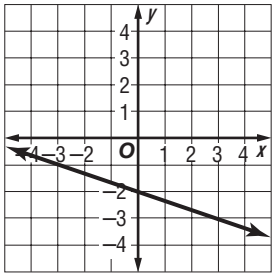
A The statement and its converse are both true.

B The statement and its converse are both false.

C The statement is true, but its converse is false.

D The statement is false, but its converse is true.

2 **ALGEBRA** Which linear function best describes the graph shown below?



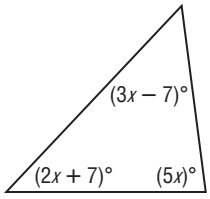
F $y = -\frac{1}{3}x - 2$

G $y = \frac{1}{3}x - 2$

H $y = \frac{1}{3}x + 2$

J $y = -\frac{1}{3}x + 2$

3 Which of the following *best* describes this triangle?



A acute isosceles

B right isosceles

C acute scalene

D right scalene

4 If $\triangle ABC$ is isosceles and $m\angle A = 94°$, which of the following *must* be true?

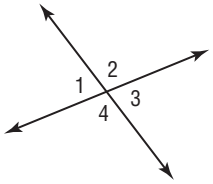
F $\angle B = 94°$

G $\angle B = 47°$

H $AB = AC$

J $AB = BC$

5 **Theorem:** If two angles are vertical angles, then they are congruent.



Tamara is proving the theorem above by contradiction. She began by assuming that vertical angles $\angle 1$ and $\angle 3$ in the diagram above are not congruent. Which theorem will Tamara use to reach a contradiction?

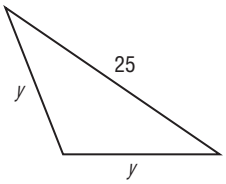
A If two angles are complementary to the same angle, the angles are congruent.

B If two angles are supplementary to the same angle, the angles are congruent.

C All right angles are congruent.

D If two angles are supplementary, the sum of their measures is 180.

6 In the figure below, y is a whole number. What is the *least* possible value for y?



7 Which of the following could be the dimensions of a triangle in units?

F 1.9, 3.2, 4

G 1.6, 3, 3.4

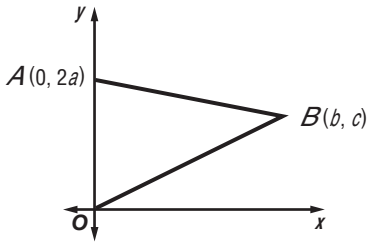
H 3, 7.2, 7.5

J 2.6, 4.5, 6

More California Standards Practice

For practice by standards, see pages CA1–CA43.

8 The diagram shows $\triangle OAB$.



What is the slope of the line that contains the altitude through vertex B of $\triangle OAB$?

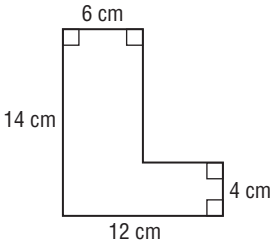
A $\frac{c-a}{b}$

B undefined

C 0

D $\frac{b}{c-a}$

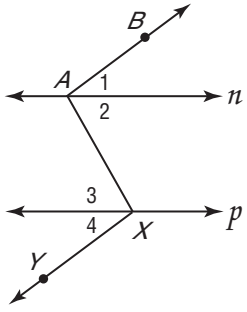
9 What is the perimeter of the figure in centimeters?



TEST-TAKING TIP

Question 9 When finding the perimeter of a figure, look for sides with measures that are missing. Find the missing measures before calculating the perimeter.

10 If line n is parallel to line m, which information would be enough to prove that $\overline{AB} \parallel \overline{XY}$?



F $m\angle 1 = m\angle 2$

G $m\angle 1 = m\angle 3$

H $m\angle 1 = m\angle 4$

J $m\angle 3 = m\angle 4$

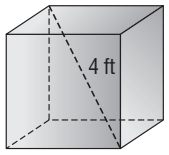
11 What is the surface area of a cube with a 4-foot diagonal?

A $\frac{4\sqrt{3}}{3}$ ft^2

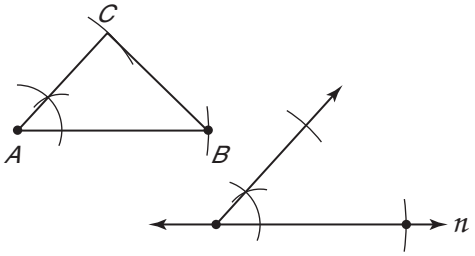
B 8 ft^2

C 32 ft^2

D 60 ft^2



12 Karl is using a straightedge and compass to do the construction shown below.



Which best describes the construction Karl is doing?

F a triangle congruent to $\triangle ABC$ using three sides

G a triangle congruent to $\triangle ABC$ using two sides and the included angle

H a triangle congruent to $\triangle ABC$ using two angles and the included angle side

J a triangle congruent to $\triangle ABC$ using two angles

Pre-AP/Anchor Problem

Record your answer on a sheet of paper. Show your work.

13 The vertices of $\triangle ABC$ are $A(-3, 1)$, $B(0, 2)$, and $C(3, 4)$. Graph $\triangle ABC$. Find the measure of each side to the nearest tenth.

a. What type of triangle is $\triangle ABC$? How do you know?

b. Describe the relationship between $m\angle A$ and $m\angle B$, $m\angle A$ and $m\angle C$, and $m\angle B$ and $m\angle C$. Explain.

NEED EXTRA HELP?													
If You Missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13
Go to Lesson or Page...	2-3	786	5-4	5-3	4-6	1-6	5-4	5-1	4-1	4-4	1-7	3-5	5-2
For Help with Standard	3.0	1A6.0	6.0	2.0	3.0	8.0	6.0	17.0	12.0	16.0	8.0	7.0	13.0